**Simplified Model for the Seismic Analysis of a Soil-Long Pile Group-Structure System**

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**ABSTRACT**

A simplified semi-analytical approach based on Thin Layered Method (TLM) and Chebyshev nested lumped-parameter model (LPM) is presented for the seismic analysis of a soil-long pile group-structure system subjected to earthquake waves. The force-displacement relationship between the unbounded soil and the long pile group is described by dynamic impedances which are obtained by the TLM. A nested lumped-parameter model based on the complex Chebyshev model is proposed to incorporate the frequency-dependent impedances with conventional governing equations for time history analysis of superstructure. The time history analysis of a 15-story superstructure supported by a 3×3 long pile group under seismic excitation is presented to show the stability and advantages of the present model.

*Keywords: Soil-structure interaction; Pile group; Impedance function; Lumped-parameter model; Chebyshev polynomials*

**1. INTRODUCTION**

Soil-structure interaction (SSI) problem has aroused much interest in earthquake engineering. Besides the experimental method and the direct method, the substructure method which enables soil and structure to be considered separately by their most suitable method is widely applied in the SSI due to its computational efficiency (Kausel 2010). Following the concept of substructure method, the procedure of analyzing the seismic response of a structure with the consideration of soil-long pile group interaction effect would be carried out in two stages:

First, determine the dynamic impedances of the pile group, which describe the reaction of the soil against piles separately from the superstructure by frequency-dependent stiffness and damping coefficients (Wang et al. 2014a; Wang et al. 2014b). The approaches to evaluate the impedance of pile-soil interaction can be classified into three categories: (1) The numerical method, such as finite element method. (2) Simplified theoretical method, such as beam-on-dynamic-Winkler model. (3) Semi-analytical method, such as thin layered method (TLM) and boundary element method. Semi-analytical method received widespread application in pile-soil interaction because it performs lower computational complexity than numerical method and higher accuracy than simplified theoretical method. Compared with boundary element method, the TLM features in analyzing the elastic wave propagation in stratified media (Zhou et al. 2015).

Second, incorporate these frequency-dependent impedances into governing equations of superstructure for the time history analysis. Since the impedances of long pile group are strongly frequency-dependent, the seismic analysis of the superstructure is generally made in the frequency domain [5]. The corresponding time history response can be obtained by using the fast Fourier transformation. However, only linear analysis of the SSI system is valid to be carried out in the frequency domain. Therefore, a variety of lumped-parameter models (LPMs), which generally consist of several mechanical components such as springs, dashpots and masses, have been proposed to represent the frequency-dependence characteristics of the impedance function in the time domain.

The semi-empirical LPMs with specified nodes were studied in early stage, which were connected by a selected arrangement of mechanical components (Wolf et al. 1986; De Barros et al. 1990; Luan et al. 1990). For convenience in constructing the systematic series expansion, a procedure to construct the consistent LPMs was proposed by Wolf (1986). Wu and Lee (2002; 2004) established different kinds of consistent LPM to represent the dynamic behaviors of shallow foundations resting on homogeneous half-space. Saitoh (2007) developed two types of LPMs using the so-called “gyromass” element. In all the aforementioned literature, the accuracy of those LPMs is significantly affected by the objective impedance functions used in the optimization. Wolf (1994) suggested that the weight adopted for the objective function for low frequencies should be higher than that for high frequencies. However, the relation between the frequency and the weight was not completely clarified. Safak (2006) used the z-transform together with a Gaussian type weighting function to develop the objective impedance function, which is expressed by a ratio of two simple polynomials. Andersen [14] approximated the impedance of hexagonal surface foundations by a ratio of two simple polynomials. To accurately represent the impedance, Andersen (2010) illustrated through a lot of numerical studies that the degree of the rational approximation should be 4 to 6 for a footing on a homogeneous half-space and 6 to 10 for a footing on a layered half-space. Generally, the stronger the dependence of the impedance on the frequency, the higher the degree of the rational approximation required to achieve a reasonable accuracy. However, too high degree of rational approximation could result in the oscillation of the results (Wang 2015). In such a case, the numerical stability of simple polynomial approximation cannot be guaranteed.

A simplified approach based on the concept of substructure method is presented for evaluating the seismic response of a soil-structure system supported by a long pile group subjected to earthquake waves. The TLM method is applied to obtain the horizontal and rocking impedances of pile group which describe the soil-pile interaction. An improved nested LPM based on the complex Chebyshev polynomials is proposed to simulate the frequency-dependent characteristics of those impedances in time domain for incorporating the governing equations of time history analysis. Due to the excellent convergence and numerical robustness of the Chebyshev polynomials, the present LPM avoid the oscillation when describe the strongly frequency-dependent impedance with high degree of polynomials. The validity of the present method is examined through a 15-story superstructure supported by a 3×3 long pile group.

**2. Soil-long pile group dynamic interaction**

The impedances of the long pile group, which can be obtained from the TLM method, show strong frequency-dependency. In order to perform the seismic response of a structure supported by a long pile group based on the substructure method in time domain, a domain-transformation method has to be introduced to deal with those frequency-dependent impedances. The general form of impedance ℜ(*ω*) of the group can be normalized with respect to its corresponding static stiffness *K*s as

ℜ(*ω*)=ℜ(*a*0)=*K*s[*K*(*a*0)+i*a*0*C*(*a*0)] (1)

It is known that the flexibility function can automatically emphasize the low-to-medium frequencies in equations without introducing any weight factor. Therefore, the dynamic flexibility function *F*(*a*0), which is normalized with respect to its corresponding static flexibility *F*s=1/*K*s, is used to describe the dynamic characteristic of the long pile group

*F*(*a*0)=*K*s× *F*d(*a*0) (2)

The normalized flexibility of the long pile group is fitted by a ratio of two complex Chebyshev polynomials

 (3)

where *s*=i*a*0/*a*0max, *a*0max is the maximum value of the approximate range of frequency. The degree of polynomial *N* is determined by minimizing the difference between the Eq. (3) and the normalized flexibility obtained from TLM via least-squares approximation. The complex Chebyshev polynomial *Tn*(i*x*) in Eq. (3) is a polynomial of i*x* of degree *n*, defined by the relation:

 (4)

where i=. Complex polynomial series {*Tn*(i*x*)} of any degree can be recursively generated from Eq. (3). The first eight complex Chebyshev polynomials are plotted in Figure 1. The coefficients *κ* and *C* in the denominator polynomials should be deliberately selected such that the following doubly asymptotic feature of *F*d(*a*0) can be ensured:

 (5)

where *σ* denotes the normalized damping coefficient at the high-frequency *a*0max. Therefore, *κ* and *C* should be respectively selected as

 (6)

 (7)

The unknown real coefficients*φn* and *ϕn* of the complex Chebyshev polynomials in Eq. (3) can be determined by the optimal approach based on the least squares fit. It is noted that the degree *N* of the polynomial should be sufficiently high to ensure a reasonable approximation to the exact flexibility. Reorganizing Eq. (3) gives the following complex polynomial fraction:

 (8)

A nested expression for Eq. (8) can be established by recursive algorithm as follows

 (9)

*j*=1

*j*=2

*j*=*N*-1

*j*=*N*

in which, , , =1, for *j*=0, 1, …, *N* and *n*=1, 2, …, *N-j*.

Wu and Lee (2004) presented the continued-fraction expansion of the impedance, which were interpreted as the frequency response function of a nested lumped-parameter model. A similar nested LPM can be introduced to represent the flexibility function *F*(*ω*), as shown in Figure 2. For the case of node *N*, the relationship between the applied load *P*0 and the resulting displacement *U*0 can be given as

 (10)

Comparing Eq. (9) and Eq. (10) with *s*=i*a*max*ωd*/*Vs*, the dimensionless coefficients of the spring *λn* and the dashpot *γn* in the Chebyshev nested LPM should be, respectively, calculated from

, , ,  for *j*=0,1,…,*N*  (11)

Since the present LPM avoids determining the poles and residues for the partial-fraction expansion of the complex Chebyshev polynomial-fraction, its parameters can be obtained by only real-number operations. Moreover, its physical configuration is independent of the problem analysed. Therefore, the present LPM can be used for establishing the dynamic governing equations of a general soil-pile group-structure system.

 

Figure 1. The first eight complex Chebyshev polynomials 



Figure 2. The nested lumped-parameter model based on the Chebyshev polynomials

**3. Seismic analysis of soil-long pile-structure system**

Consider an *N*s-story superstructure supported by a long pile group embedded in the homogeneous soil medium as shown in Figure 3. As the physical configuration of the present LPM is not dependent on a given example, it can be assumed that Chebyshev nested LPMs with *N*h degrees of freedom in the horizontal direction and *N*r degrees of freedom in rocking direction are adopted to simulate the dynamic interaction between soil and long pile group in the time domain. Each story of the superstructure has a lumped mass *m*s*i* and a mass moment of inertia *I*s*i*. *m*0 and *I*0 are the mass and the mass moment of inertia of the pile group. *k*s*i* is the superstructure inter-story stiffness. *hi* is the story height which measured from the centre of the pile cap to the *i*-th story. The displacement of *i*-th story with respect to the pile cap is denoted as *u*s*i*. The displacements of the pile cap and all horizontal degrees of the LPM with respected to the fixed end of the LPM are denoted as *u*f and *u*h*i*, respectively.

When the system is subjected to a ground horizontal earthquake wave with acceleration g, the dynamic equations of equilibrium for the total system based on D’Alembert’s principle can be written in the following matrix form

(12)

Here, the first *N*s governing equations in Eq. (12) are established according to the transverse equilibrium of each story. The (*N*s+1)-*th* governing equation is established according to the transverse equilibrium of the superstructure-pile group system. The (*N*s+2)-*th* to (*N*s+*N*h +1)-*th* governing equations are established according to the transverse equilibrium of each node in horizontal LPM. The (*N*s+*N*h+2)-*th* governing equation is established according to the rocking equilibrium of the superstructure-pile group system. The (*N*s+*N*h+3)-*th* to (*N*s+*N*h+*N*r+2)-*th* governing equations are established according to the rocking equilibrium of each node in rocking LPM.

In order to estimate the seismic response of the system, the solution of Eq. (12) can be obtained by Wilson *θ*-method which is unconditionally stable in linear analysis and can also be employed in nonlinear analyses. Moreover, the natural frequency of the system can be analysed by the complex model theory.



Figure 3. Simplified model for soil-long pile group-superstructure system

**4. Numerical Example**

This example shows the applicability and stability of the present simplified approach for seismic analysis of a 15-story superstructure rested on a 3×3 long pile group under 200gal Taft Earthquake wave. The time history and response spectrum of Taft wave are shown in Figure 4. The mass, stiffness and moment of inertia at each story of superstructure are 25000*kg*, 5×107*N*/*m* and 1.5×106 *kg*⋅*m*2, respectively. The height between adjacent floors is 3.5*m*. The density, shear velocity, Possion’s ratio and damping ratio of the soil are 1400 *kg*/*m*2, 100*m*/*s*, 0.4 and 0.05, respectively. The diameter, length, density, Young’s modulus and Possion’s ratio of piles in the group are 1*m*, 20*m*, 2000*kg*/*m*2, 3.92×109*N*/*m*2 and 0.25. The distance between the centers of the adjacent piles is 5*m*.

The horizontal and rocking impedances, which describe the soil-pile dynamic interaction, are obtained by the Chebyshev nested LPM. The present LPM give satisfactory simulations to horizontal and rocking dynamic impedances of 3×3 long pile group as shown in Figs. 5 and 6, respectively. Figure 5 corresponding to complex Chebyshev polynomials of degrees *N*=2, 3 and 4, as well as *N*=4, 8 and 12 in Figure 6, is presented for illustrating different degrees of accuracy. It can be seen from Figs. 8 and 9 that the difference between the solution in time domain and that in frequency domain decreases when increasing the degree *N*. As shown in Figs. 5 and 6, the Chebyshev nested LPMs using polynomial degree *N*=4 for horizontal impedance and *N*=12 for rocking impedance can satisfactorily simulate the frequency-dependent characteristic of foundation impedances in the time domain. The corresponding coefficients of springs and dashpots in the LPM are shown in Table 1. Figure 7 shows the time-history response of the top story of the superstructure both including and excluding the soil-long pile group interaction. It is shown in Figure 7 that the peak acceleration and the velocity of the top story for the case of soft soil is significantly reduced as compared with the fixed-foundation system, respectively.

Table 1. The coefficients of springs and dashpots in the nested LPM of a 3×3 long pile group

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Coefficients *λi*** | **Horizontal coefficients** | **Rocking coefficients** | **Coefficients *γi*** | **Horizontal coefficients** | **Rocking coefficients** |
| *λ*0 | 1 | 1 | *γ*0 | 0.17 | 0.9 |
| *λ*1 | 75.19 | -0.508 | *γ*1 | 2.25 | -1.38 |
| *λ*2 | -0.707 | 8.68 | *γ*2 | -0.290 | 1.35 |
| *λ*3 | 0.621 | -0.270 | *γ*3 | 2.49 | -2.77 |
| *λ*4 | 1.41 | -3.90 | *γ*4 | -3.27 | 3.78 |
| *λ*5 | - | 4.56 | *γ*5 | - | -2.66 |
| *λ*6 | - | 0.985 | *γ*6 | - | 1.61 |
| *λ*7 | - | -0.118 | *γ*7 | - | -0.259 |
| *λ*8 | - | 0.284 | *γ*8 | - | 0.385 |
| *λ*9 | - | 0.0240 | *γ*9 | - | 0.0513 |
| *λ*10 | - | -0.0492 | *γ*10 | - | -0.126 |
| *λ*11 | - | 0.00151 | *γ*11 | - | 0.00020 |
| *λ*12 | - | -0.00151 | *γ*12 | - | -0.169 |

 

Figure 4. Time history and response spectrum of acceleration under Talf ground motion

|  |  |
| --- | --- |
|  |  |
| 1. Stiffness | 1. Damping |
| Figure 5. Normalized horizontal impedances of a 3×3 long pile group | |

|  |  |
| --- | --- |
|  |  |
| 1. Stiffness | 1. Damping |
| Figure 6. Normalized rocking impedances of a 3×3 long pile group | |

|  |  |
| --- | --- |
|  |  |
| (a) Acceleration response | (b) Velocity response |
| Figure 7. Seismic response of the top story of superstructure | |

**5. Conclusions**

Following the concept of the substructure method, a simplified analytical approach is proposed for the seismic analysis of a long pile group supported superstructure subjected to earthquake waves. The dynamic interaction between the soil and the long pile group is evaluated by the TLM in the frequency domain. A Chebyshev nested LPM is developed to transform the frequency-dependent impedance into time domain for history analysis of a superstructure with consideration of the soil-long pile interaction effect. The highlights of the simplified analytical approach can be shown through the numerical examples, as summarized: 1) The present LPM can express the strongly frequency-dependent impedance of long pile group through a small number of elements and reduce the oscillation in the approximate solutions. 2) The present LPM contains no mass, the input excitation can be directly applied and incorporated into the existing time history dynamic analysis programs without any modification.

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