**OPTIMUM PROPERTIES OF SEISMIC ISOLATION SYSTEMS**

**IN HIGHWAY BRIDGES TO MINIMIZE ISOLATOR DISPLACEMENTS OR SUBSTRUCTURE FORCES**

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**ABSTRACT**

In this study, closed form equations as functions of the isolator, bridge and ground motion properties are formulated to calculate the optimum characteristic strength, Qd and post-elastic stiffness, kd, of the isolator to minimize the maximum isolator displacement (MID) and force (MIF) for seismic isolated bridges (SIBs). For this purpose, first, sensitivity analyses are conducted to identify the bridge, isolator and ground motion parameters that affect the optimum values of Qd and kd. Next, for the identified parameters, nonlinear time history analyses of typical SIBs are conducted to determine the optimum values of Qd and kd for a wide range of values of the parameters. Next, nonlinear regression analyses of the available data are conducted to obtain closed form equations for the optimum values of Qd and kd to minimize the MID and MIF. The equations are then simplified for various site soil conditions. It is observed that the optimum Qd and kd are highly dependent on the site soil condition. However, the effect of the bridge substructure stiffness on the optimum Qd and kd and the effect of the structural or supplemental damping on the optimum kd are found to be negligible. Furthermore, the optimum Qd is found to be a linear function of the peak ground acceleration.

*Keywords:* *seismic isolation, bridge, optimization, site soil*

**1. INTRODUCTION**

The force-displacement hysteresis of most commercially available isolators is generally idealized as bilinear for design purposes. A typical bilinear force-displacement hysteresis of an isolator and a typical isolated bridge substructure are shown in Figs. 1(a) and (b). In the figures, Qd is the characteristic strength, ku is the elastic stiffness, kd is the post-elastic stiffness, Fy and uy are respectively the yield force and displacement and Fi and ui are respectively the maximum (or design) force and displacement of the isolator. The characteristic strength, Qd and the post-elastic stiffness, kd, are the main isolator parameters that affect the behavior of a SIB for a given ground motion with specific frequency characteristics and intensity (Dicleli & Buddaram 2006). Thus, the optimal selection of these isolator parameters based on minimizing the MID and MIF will result in an economical design of the SIB. Several research studies have been conducted to identify the optimal characteristic properties of isolators or yielding systems for the seismic design of structures (Veletsos et al. 1965, Park & Otsuka 1999, Iemura et al. 2007). However, none of these research studies provide simple yet effective equations to calculate the optimal isolator properties. Therefore, it is clear that closed form equations as functions of the properties of the bridge, isolator and frequency characteristics and intensity of actual ground motions are still needed to obtain the optimum values of the isolator characteristic parameters, Qd and kd based on minimizing the MID and MIF. Such equations may be used by the bridge engineering community to select proper values of Qd and kd for the economical seismic design of SIBs.

**U**

F

i

U

i

Q

d

k

d

1

k

u

F

y

U

y

**F**

**Superstructure**

**Isolator**



**Substructure**

**(a) (b)**

Figure 1. (a) Idealized hysteresis loop of a typical isolator, (b) Typical seismic-isolated bridge substructure.

**2. RESEARCH OBJECTIVE AND METHODOLOGY**

The main objective of this research study is to formulate closed form equations as functions of the isolator, bridge and ground motion properties to calculate the optimum characteristic strength, *Qd* and post-elastic stiffness, *kd*, of the isolator to minimize the MID and MIF. Using the developed equations it will be possible to select the proper isolator properties that will result in an economical SIB design. To achieve the above stated objective, first, sensitivity analyses are conducted to investigate the effects of several bridge, isolator and ground motion parameters on the optimum values of *Qd* and *kd*. From these sensitivity analyses, the parameters that affect the optimum values of *Qd* and *kd* are identified. Next, for each one of the identified parameters, nonlinear time history (NLTH) analyses of simplified structural models representative of typical SIBs are conducted to determine the optimum values of *Qd* and *kd* for an assumed range of values of the parameter under consideration. Furthermore, for each assigned value of the parameter under consideration, the NLTH analyses are repeated until the optimum values of *Qd* and *kd* are correctly identified. Next, the available data is plotted as a function of some dimensionless parameters proposed by Makris& Black (2004a, 2004b). Nonlinear regression analyses of the plotted data are then conducted to obtain closed form equations for the optimum values of *Qd* and *kd*, to minimize the MID and MIF. The obtained closed form equations are then verified using a suite of ground motions different than that used for the development of the same equations.

**3. GROUND MOTIONS AND PARAMETERS CONSIDERED**

Two sets of ground motions are used. The first set involves a suite of 15 earthquakes with Ap/Vp ratios ranging between 5.50 s-1 and 21.5 s-1 presented in Table 1 (No.: 1-15). These ground motions are used for the formulation of the optimum values of Qd and kd. The second set involves a suite of five earth-quakes with Ap/Vp ratios ranging between 4.70 s-1 and 23.6 s-1 presented also in Table 1 (No.: 16-20). These ground motions are used for the verification of the equations developed to calculate the optimum values of Qd and kd.

Several parameters are considered for the sensitivity analyses to study the effect of the bridge, isolator and ground motion properties on the optimum values of Qd and kd. The parameters that affect the optimum values of Qd and kd are then used in the optimization procedure. The parameters that are used in the sensitivity analyses are categorized into three groups as those representing the bridge, isolator and ground motion properties. The bridge properties are represented by the superstructure mass, m, substructure stiffness, ks and structural or supple-mental damping, ζ. The superstructure mass tributary to a typical isolator is taken as a constant equal to 204 tons (2000 kN weight). The substructure stiffness is varied between 200 kN/cm (very flexible) and 6400 kN/cm (very stiff). This stiffness range corresponds to 10 to 320 times the 20 kN/cm post elastic stiffness of the isolator used in the analyses while studying the effect of the substructure stiffness on the optimum values of Qd and kd. The structural and/or supplemental damping is varied between 0% and 8% of critical. The isolator properties are represented by the characteristic strength, Qd and post-elastic stiffness, kd. The isolator’s elastic stiffness is not considered as a parameter in this study since its effect on the performance of SIBs has been found to be negligible earlier by Dicleli and Buddaram (2006).

Table 1. Important features of the earthquake records used in the analyses.

|  |  |  |  |
| --- | --- | --- | --- |
| **No** | **Earthquake** | **Station/Component** | **Ap/Vp** |
| **1** | San Fernando, 1971 | 8244 Orion Blvd. / 180º | 5.5 |
| **2** | Imperial Valley, 1940 | El Centro / 180º | 5.8 |
| **3** | Loma Prieta, 1989 | Oakland Outer Wharf / 0º | 6.1 |
| **4** | Loma Prieta, 1989 | Oakland Outer Wharf / 270º | 7.2 |
| **5** | Northridge, 1994 | Arleta and Nordhoff Fire Station / 90º | 8.4 |
| **6** | Kern County, 1952 | Taft Lincoln Tunnel / 69º | 9.7 |
| **7** | Imperial Valley, 1940 | El Centro / 270º | 10.6 |
| **8** | Santa Barbara, 1978 | 283 Santa Barbara Courthouse / 222º | 12.2 |
| **9** | Coalinga, 1983 | 36227 Parkfield - Cholame 5W / 270º | 13.4 |
| **10** | Northridge, 1994 | Santa Monica City Hall Grounds / 0º | 14.6 |
| **11** | Whitter Narrows, 1987 | 24401 San Marino, SW Academy / 360º | 15.6 |
| **12** | Whitter Narrows, 1987 | 90079 Downey Birchdale / 90º | 17.4 |
| **13** | San Fernando, 1971 | Pacoima Dam. / 196º | 18.4 |
| **14** | Northridge, 1994 | Santa Monica City Hall Grounds / 90º | 20.7 |
| **15** | Parkfield, 1966 | Cholame, Shandon / 40º | 21.5 |
| **16** | Borrego Mount., 1968 | Hollywood Storage Lot / 180º | 4.7 |
| **17** | Kobe, 1995 | FUK / 0º | 7.7 |
| **18** | Friuli, Italy, 1976 | Conegliano / 0º | 13.6 |
| **19** | NW California, 1941 | Ferndale City Hall / 45º | 16.7 |
| **20** | Morgan Hill, 1984 | San Francisco Int. Airport, / 90º | 23.6 |

**4. DIMENSIONLESS PARAMETERS USED IN THE DEVELOPMENT OF THE EQUATIONS**

Using formal dimensional analysis, Makris & Black (2004a, 2004b) proposed four dimensionless terms to describe the response of a structural system with bilinear force-deformation relationship in relation to the characteristics of a pulse type excitation with a period, Tp. Two of the proposed terms; Π2=Qd/mAp, and Π4= Td/Tp, are used in this study to represent the analyses results where Td is the period of the bridge based on the post-elastic stiffness, kd, of the isolator and all the other variables are as described before. Since seismic ground motions with various frequency characteristics are considered in the presented study, the term, Tp, is replaced by the dominant period of the ground motion given as Tg= 2π/(Ap/Vp) (Kramer 1996). The term, Π2=Qd/mAp represents the ratio of the characteristic strength of the isolator to the seismic inertial force of a rigid bridge superstructure. Using this term eliminates the number of independent variables used in the analyses.

**5. OPTIMIZATION PROCEDURE**

The calculation of the optimum values of Qd and kd to minimize the MIF and MID consists of three sets of analyses. The first two sets of analyses are performed to obtain the optimum values of Qd to minimize the MIF and MID respectively. The third set of analyses is performed to obtain the optimum values of kd to minimize the MID.

To calculate the optimum values of Qd to minimize the MIF for the range of parameters and ground motions considered in this study, first, the MIF is determined for a wide range of presumed Qd values. Next, the bisection search method (Chapra & Canale 2002) is used between the data points before and after the apparent minimum value of MIF within the MIF-Qd plot, to determine the actual minimum MIF and the corresponding optimal value of Qd. This required repeating the NLTH analyses of the simplified bridge model until convergency is reached for the optimum values of Qd. This procedure is repeated for the range of parameters and ground motions considered in this study to obtain a wide range of data giving the optimal Qd values. Similar procedures are also used to obtain the optimum values of Qd and kd corresponding to minimum MID.

**6. EFFECT OF VARIOUS PARAMETERS ON OPTIMUM QD**

In this section, sensitivity analyses are conducted to identify the parameters that affect the optimum value of Qd, based on minimizing the MIF and MID. For this purpose, iterative NLTH analyses of simplified structural models of SIBs are conducted. In the analyses, a single bridge substructure and an isolator supporting a tributary bridge superstructure is considered as shown in Figure 1(b). The superstructure is assumed to have infinite in-plane rigidity as normally done in the design of most SIBs. The bridge, isolator and ground motion parameters used in the analyses are; m=204 ton (W=2000 kN), ζ=0, ku=200 kN/cm, kd=20 kN/cm, ks/kd=160, Ap=0.6g and Ap/Vp=5.8, 12.2 and 21.5 s-1 (i.e. the analyses are repeated for three ground motions). In the sensitivity analyses, while the parameter under consideration is varied, the rest of the parameters are assigned the values reported above. The sensitivity analyses results for each parameter are given below.

***6.1 Effect of Peak Ground Acceleration***

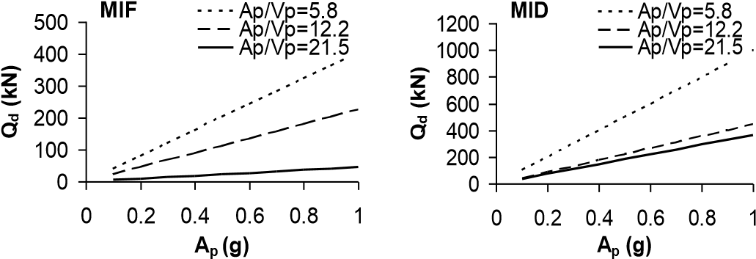
In this section, the effect of the peak ground acceleration, Ap on the optimum value of Qd is investigated by varying the peak ground acceleration between 0.1g and 1.0g while keeping the other parameters constant. The sensitivity analyses results are presented in Figs. 2(a) and (b) for three ground motions with Ap/Vp =5.8, 12.2 and 21.5 s-1 in the form of optimum Qd versus Ap plots based on minimizing the MIF and MID respectively. As observed from the figures, the optimum value of Qd is linearly proportional to Ap. Knowing this, the optimum value of Qd is normalized with respect to mAp as suggested by Makris & Black (2004a) and re-plotted in Figs. 2(c) and (d). As observed from the figures, the Qd/mAp ratio is independent of the peak ground acceleration, Ap. Thus, for the remainder of the study, the peak ground acceleration is not considered as an independent parameter in the development of the closed form equations to calculate the optimum values of Qd based on minimizing MIF and MID. Instead the Qd/mAp ratio is used in the optimization procedure. The dimensionless term, Qd/mAp, is also used in the presentation of the analyses results for the remainder of the study.

***6.2 Effect of Ap/Vp Ratio of Ground Motion Effect***

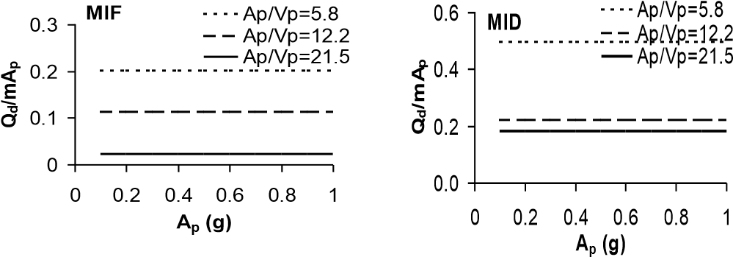
T in this section, the effect of the Ap/Vp ratio of the ground motion on the optimum value of Qd is investigated. The sensitivity analyses results presented in Figs. 2(a), (b), (c) and (d) reveal that, the optimum value of Qd/mAp ratio or Qd is highly dependent on the Ap/Vp ratio of the ground motion. That is, for each Ap/Vp ratio, a different relationship between Qd and Ap is obtained. In general, the sensitivity analyses results revealed that the optimum value of Qd increases as the Ap/Vp ratio of the ground motion decreases (Figs. 2(a) and (b)). Thus, the Ap/Vp ratio of the ground motion must be included as a parameter in the development of closed form equations to calculate the optimum values of Qd based on minimizing the MIF and MID.

***6.3 Effect of Post-Elastic Stiffness of the Isolator***

In this section, the effect of the post-elastic stiffness, kd, of the isolator on the optimum value of Qd is investigated. For this purpose, NLTH analyses of simplified structural models of SIBs with isolators having kd ranging between 1.25 and 40 kN/cm are conducted for three ground motions with Ap/Vp= 5.8, 12.2 and 21.5 s-1 scaled to Ap = 0.6g to determine the optimum values of Qd based on minimizing the MIF and MID. The analyses results are presented in Figs. 3(a) and (b). As observed from the figures, the optimum Qd/mAp ratio varies as a function of kd. Thus, kd must be included as a parameter in the development of closed form equations to calculate the optimum values of Qd based on minimizing the MIF and MID.

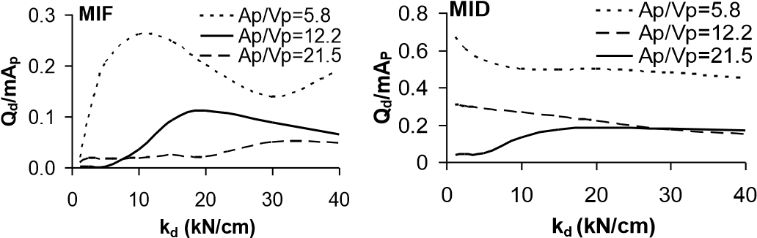


**(a) (b)**



**(c) (d)**

Figure 2. Effect of various parameters on optimum Qd (a) Ap for MIF (b) Ap for MID, (c) Ap for MIF using Qd/mAp ratio, (d) Ap for MID using Qd/mAp ratio.



**(a)**  **(b)**

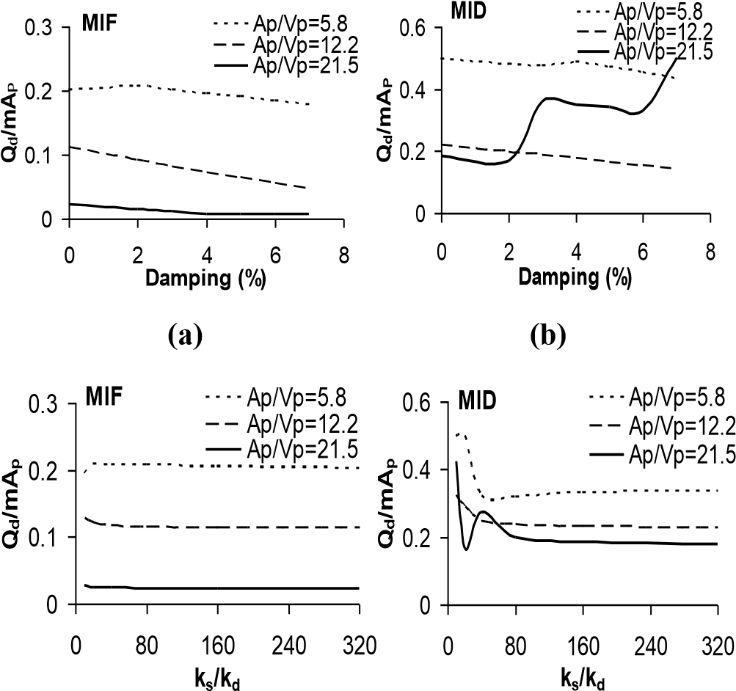
Figure 3. Effect of various parameters on optimum Qd (a) kd for MIF, (b) kd for MID.

***6.4 Effect of Structural/Supplemental Damping***

In this section, the effect of structural/supplemental damping on the optimum value of Qd is investigated. For this purpose, NLTH analyses of simplified structural models of SIBs with structural/supplemental damping ratios ranging between 0% and 7% are conducted for three ground motions with Ap/Vp= 5.8, 12.2 and 21.5 s1 scaled to Ap = 0.6g to determine the optimum values of Qd based on minimizing the MIF and MID. The analyses results are presented in Figs. 4(a) and (b). As observed from the figures, the optimum Qd/mAp ratio varies as a function of the damping ratio. Thus, damping ratio must be included as a parameter in the development of closed form equations to calculate the optimum values of Qd based on minimizing the MIF and MID.

***6.5 Effect of Substructure Stiffness***

In this section, the effect of the substructure stiffness on the optimum value of Qd is investigated by changing, in the structural model, the ratio, ks/kd, of the lateral stiffness of the substructure to the post-elastic stiffness of the isolator between 10 (very flexible) and 320 (very stiff). The analyses results are presented in Figure 4(c) and (d). As observed from the figures, the optimum Qd/mAp ratio or Qd does not vary as a function of ks/kd (i.e. the curves remain nearly flat for the range of ks/kd ratios considered) except for the cases where the substructure is very flexible (ks/kd < 40).





**(**

**c**

**)**



**(**

**d**

**)**

Figure 4. Effect of various parameters on optimum *Qd* (a)Damping for MIF, (b)Damping for MID (c) Substructure stiffness for MIF, (d) Substructure stiffness for MID.

**7. EFFECT OF VARIOUS PARAMETERS ON OPTIMUM KD**

In this section, the effect of various parameters on the optimum value of kd, based on minimizing the MID, is investigated via sensitivity analyses. The bridge, isolator and ground motion parameters used in the analyses are; m=204 ton (W=2000 kN), ζ=0, ku=200 kN/cm, Qd=100 kN, ks=3200 kN/cm, Ap=0.6g and Ap/Vp=5.8, 12.2 and 21.5 s-1 (i.e. the analyses are repeated for three ground motions). In the sensitivity analyses, while the parameter under consideration is varied, the rest of the parameters are assigned the values reported above. The sensitivity analyses results for each parameter are given below.

***7.1 Effect of Peak Ground Acceleration and Characteristic Strength***

As discussed earlier, the dimensionless term, Qd/mAp represents the ratio of the characteristic strength of the isolator to the seismic inertial force of a rigid bridge superstructure and eliminates the need for considering the peak ground acceleration and the characteristic strength of the isolator independently. Thus, the Qd/mAp ratio is used in the sensitivity analyses to study the combined effects of Ap and Qd on the optimum value of kd. The analyses results are presented in Figs 5(a) and for three ground motions with Ap/Vp =5.8, 12.2 and 21.5 s-1 in the form of optimum kd versus Qd/mAp plots based on minimizing the MID. As observed from the figure, the optimum value of kd varies as a function of the Qd/mAp ratio. Thus, the Qd/mAp ratio must be included as a parameter in the development of the closed form equation to calculate the optimum values of kd based on minimizing the MID.

***7.2 Effect of Ap/Vp Ratio of Ground Motion***

In this section, the effect of the Ap/Vp ratio of the ground motion on the optimum value of kd is investigated by varying the Ap/Vp ratio between 5.5 and 21.5 s-1 while keeping the other parameters constant. The sensitivity analyses results are presented in Figure 5(b) in the form of an optimum kd versus Ap/Vp ratio plot based on minimizing the MID. As observed from the figure, the optimum value of kd is highly dependent on the Ap/Vp ratio of the ground motion. In fact, the optimum value of kd increases as the Ap/Vp ratio of the ground motion decreases. Thus, the Ap/Vp ratio of the ground motion must be included as a parameter in the development of closed form equations to calculate the optimum values of kd based on minimizing the MID.

0

50

100

150

200

0.05

0.1

0.15

0.2

**Q**

**d**

**/mA**

**p**

**k**

**d**

**(kN/cm)**

Ap/Vp=5.8

Ap/Vp=12.2

Ap/Vp=21.5

0

100

200

5.5

9.5

13.5

17.5

21.5

**A**

**p**

**/V**

**p**

**k**

**d**

**(kN/cm)**

**(a) (b)**

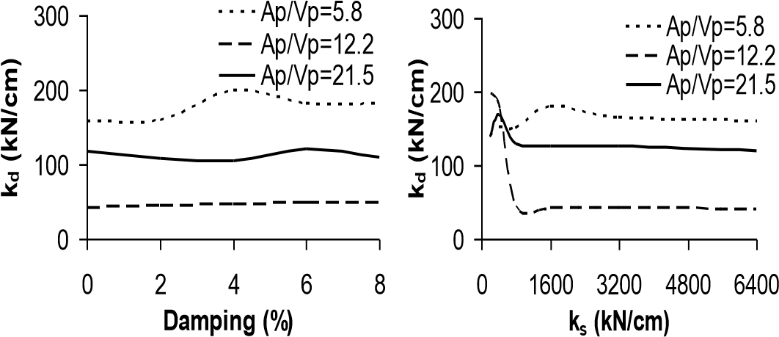
Figure 5. Effect of various parameters on optimum kd based on minimizing the MID (a) Qd/mAp ratio (b) Ap/Vp ratio.

***7.3 Effect of Structural/Supplemental Damping***

In this section, the effect of structural/supplemental damping on the optimum value of kd is investigated. For this purpose, NLTH analyses of simplified structural models of SIBs with structural/supplemental damping ratios ranging between 0% and 8% are conducted for three ground motions with Ap/Vp =5.8, 12.2 and 21.5 s1 scaled to Ap = 0.6g to determine the optimum values of kd based on minimizing the MID. The analyses results are presented in Figure 6(a). As observed from the figures, the optimum value of kd does not vary significantly as a function of the damping ratio. Thus, damping ratio need not be included as a parameter in the development of closed form equations to calculate the optimum values of kd based on minimizing the MID.

***7.4 Effect of Substructure Stiffness***

In this section, the effect of the substructure stiffness on the optimum value of kd is investigated by changing, in the structural model, the lateral stiffness, ks, of the substructure between 200 kN/cm (very flexible) and 6400 kN/cm (very stiff). The analyses results are presented in Figure 6(b). As observed from the figure, the optimum value of kd does not vary as a function of ks (i.e. the curves remain nearly flat) except for the cases where the substructure is very flexible.



**(a) (b)**

Figure 6. Effect of various parameters on optimum kd based on minimizing the MID (a) Damping, (b) Substructure stiffness.

**8. FORMULATION OF THE OPTIMUM CHARACTERISTIC STRENGTH**

In view of the sensitivity analyses results, the optimum values of Qd based on minimizing the MIF and MID are calculated for a wide range of values of the parameters that are found to affect the optimum value Qd. The analyses results for the optimum values of Qd based on minimizing the MIF and MID are presented respectively in Figure 7(a) and (b) in the form of Qd/mAp versus Td/Tg plots. These graphs and similar plots of Qd/mAp versus damping are used in the formulation of the optimum Qd based on minimizing the MIF and

MID via regression analyses. Thus, the optimum Qd-F is obtained as;

(1)

Note that the post elastic period of the SIB, Td and the dominant period of the ground motion, Tg, in Eq. (1) are expressed as;

(2)

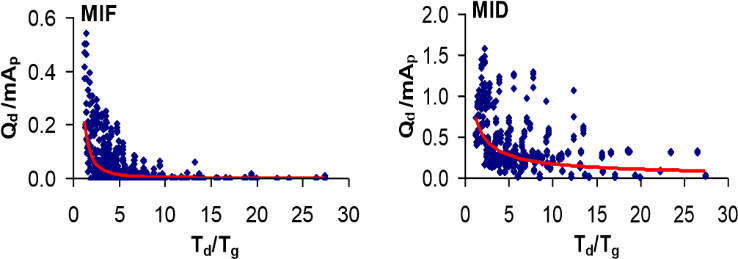
(3)

Substituting Eqns. (2) and (3) into Eqn. (1) and rearranging, the optimum Qd-F based on minimizing the MIF is obtained as;

(4)

The optimum characteristic strength Qd-D based on minimizing the MID is also obtained as;

(5)





**(**

**a**

**)**



**(**

**b**

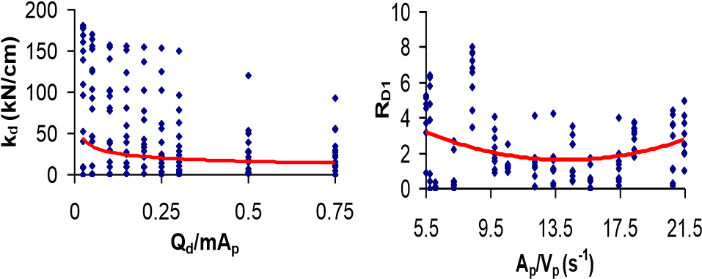
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Figure 7. (a) Variation of the optimum Qd/mAp ratio as a function of Td/Tg ratio based on minimizing the MIF, (b) Variation of the optimum Qd/mAp ratio as a function of Td/Tg ratio based on minimizing the MID.

**9. FORMULATION OF THE OPTIMUM POST-ELASTIC STIFFNESS**

In light of the sensitivity analyses results, the optimum values of kd based on minimizing the MID are calculated for the assumed range of values of the parameters that are found to influence the optimum value of kd. The analyses results for the optimum values of kd based on minimizing the MID are presented in Figure 8(a) in the form of kd versus Qd/mAp plot. This graph and the plot in Figure 8(b) are used in the formulation of the optimum kd based on minimizing the MID. The following equation that may be used to calculate the optimum values of kd to achieve the smallest possible MID is obtained via regression analyses;

(6)



**(a) (b)**

Figure 8. (a) Variation of the optimum kd as a function of Qd/mAp ratio based on minimizing the MID, (b) Ratio of the optimum kd based on minimizing the MID as a function of the Ap/Vp ratio.

**10. VERIFICATION OF THE DEVELOPED EQUATIONS**

The developed equations are verified using a suite of five ground motions different than those used for their development (Record # 16-20 in Table 1). The comparison of the optimum Qd and kd values obtained from the developed equations (Eqns. (4), (5) and (6)) with those obtained from iterative NLTH analyses are presented as a function of the Ap/Vp ratio of the ground motions in Figure 9 for various isolator properties and structural/supplemental damping ratios. As observed from the plots of Figure 9, although some differences between the analytical and NLTH analyses results are noted at specific Ap/Vp points, in general, the overall variation of the developed equations as a function of the Ap/Vp ratio of the ground motions agrees well with the analysis results.

**MID, ζ=0, k = 20 kN/cm**

0

200

400

600

800

**Q**

**d-F**

**kN)**

**(**

Analysis

Eqn. (7)

**MIF, ζ=0, k**

**d**

**=**

**20 kN/cm**

0

400

800

1200

**Q**

**d-D**

**(**

**)**

**kN**

Analysis

Eqn. (8)

**d**

4 8 12 16 20 24 4 8 12 16 20 24

**Ap/Vp (s-1) Ap/Vp (s-1)**

**(a) (b)**

**Q**

**d**

**/W=0.05**

0

50

100

150

200

4

8

12

16

20

24

**A**

**p**

**/V**

**p**

**(**

**s**

**-1**

**)**

**k**

**d**

**(**

**kN/cm**

**)**

Analysis

Eqn. (11)

**(c)**

Figure 9. Comparison of the optimum isolator characteristic parameters (Qd-F, Qd-D, kd) obtained from the developed equations with those obtained from NLTH analyses (a) Eqn. (4), (b) Eqn. (5), (c) Eqn. (6)

**11. CONCLUSIONS**

In this study, closed form equations as functions of the isolator, bridge and ground motion properties are formulated to calculate the optimum characteristic strength, Qd and post-elastic stiffness, kd, of the isolator to minimize the MID and MIF for SIBs. It is found that, the effect of the bridge substructure stiffness on the optimum Qd and kd and the effect of the structural/supplemental damping on the optimum kd are negligible. The optimum Qd and kd are found to be highly dependent on the frequency characteristics of the ground motion and the peak ground acceleration.

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