**Improved seismic base isolation combined with fluid inerter and tuned mass damper**

**DOI 10.37153/2686-7974-2019-16-176-188**

Dario DE DOMENICO[[1]](#footnote-1), Predaricka DEASTRA[[2]](#footnote-2), Giuseppe RICCIARDI[[3]](#footnote-3), Neil D. SIMS[[4]](#footnote-4), David J. WAGG[[5]](#footnote-5)

**Abstract**

The aim of this contribution is to present an improved seismic base isolation combined with fluid inerter and tuned mass damper (TMD). This combined system exploits the advantageous properties of the fluid inerter device for an optimized structural control of base-isolated buildings. In particular, this device is based on the inertia of a fluid passing through a helical tube surrounding a cylinder equipped with piston. Such inertia induces a beneficial mass-amplification effect that improves the performance of the attached TMD, resulting in the so-called tuned mass damper inerter (TMDI). Moreover, the fluid inerter has some inherent damping properties due to the pressure drops occurring in the helical channels induced by the fluid viscosity. The parameters of the fluid inerter are determined through a numerical procedure aimed to optimize the structural control of base-isolated structures and a sensitivity analysis is also performed. Finally, a practical procedure for converting the optimal parameters obtained from the optimization procedure into a realistic fluid inerter device is presented and applied to a simple case study.

*Keywords: Fluid inerter; Seismic base isolation; Tuned mass damper; Earthquake protection; Optimal design.*

**1. Introduction**

Introduced by Smith in 2002 (Smith 2002), the inerter is gaining popularity in the field of vibration control. The inerter is a two-terminal device whose resisting force is proportional to the relative acceleration of its terminals, analogous to the way the spring and dashpot offer a resisting force proportional to the relative displacement and velocity. Considering this peculiarity, the inerter contributes to the inertial properties of the system to which it is connected, acting as an additional, apparent mass. What is more, the inerter has an inherent mass amplification effect (also called “inertance”) that can be designed to be much higher than the actual physical mass of the device: as an example, a commercially available inerter tailored for vehicle suspensions produces an inertance of around 75 kg with a self-weight of just 2 kg (Smith 2002).

The mass amplification effect of the inerter can be usefully employed to improve the performance of tuned mass damper (TMD) systems in earthquake engineering applications. When the inerter is incorporated in a TMD system, a lower mass and more effective alternative to the TMD is obtained, called tuned mass damper inerter (TMDI) (Marian and Giaralis 2014) or tuned inerter damper (Lazar et al. 2014). Typical inerter realizations include rack-and-pinion systems and ball screw mechanisms. More recently, a hydraulic realization of the inerter, called the fluid inerter, was patented by Smith and collaborators (Swift et al. 2013). The helical fluid inerter is formed by a piston moving within a cylinder that contains a viscous fluid. Instead of forcing the fluid to pass through a small orifice, like in fluid viscous dampers (De Domenico et al. 2019), the linear movement of the piston makes the fluid pass through a helical tube surrounding the cylinder. This flow generates an inertial force due to the moving mass of the fluid. Fluid inerters may have advantages over mechanical inerters, as they reduce ratcheting, backlash and friction phenomena (Gonzalez-Buelga et al. 2017). Additionally, the fluid inerter has an inherent damping due to pressure drops occurring in the helical channels related to the fluid viscosity (Smith and Wagg 2016), which is convenient for structural control purposes.

The aim of this contribution is to present an improved seismic base isolation combined with a TMDI that exploits the aforementioned advantageous properties of the fluid inerter. The motivation of this complex structural control scheme is related to the need to reduce the large displacements concentrated at the isolation level in base-isolated buildings. While the combination of the base isolation with a TMD has been already investigated in the literature (see e.g. Taniguchi et al. 2008; De Domenico and Ricciardi 2018d), enhanced versions with the TMDI have been explored by the authors very recently (De Domenico and Ricciardi 2018a, 2018b, 2018c, 2018e; De Domenico et al. 2018b), but confined to mechanical inerters. Thus, this contribution aims to extend the previous studies by incorporating the fluid inerter in place of the rack-and-pinion mechanical inerter. The parameters of the fluid inerter, including inertance and nonlinear damping, are first determined through a numerical procedure aimed to optimize the structural control of the base-isolated structure. Response history analysis is finally carried out considering 44 earthquake ground motions to assess the seismic performance of the improved base-isolated system equipped with TMD and fluid inerter.

**2. Helical fluid inerter: modelling and experiments**

Extensive description of the helical fluid inerter was provided in the paper by Swift et al. (2013) and in the international patent by Glover et al. (2009). Here, we recall the basics of the mechanical behavior of this device and complement the previous explanations with some additional considerations based on experimental findings. Reference is made to the schematic diagram shown in Figure 1. The device consists of a piston forcing a fluid to flow into a helical channel. The moving mass of the fluid gives rise to an inertia force that is reasonably expressed by the following formula

 (1)

which is proportional to the relative accelerations between the two terminals of the device . The constant  has dimensions of mass [kg] and is termed inertance, while  approximates the amount of fluid mass moving within the helical tube. By adjusting the  ratio, where  represents the cross-sectional area of the cylinder and  that of the helical channel, the value of the inertance  can be designed through simple geometrical considerations. Obviously, Eq. (1) is a simplified model. In reality, there are other nonlinear damping contributions to the resisting force of the device that involve the physical characteristics of the internal fluid (e.g., viscosity and density), which cause certain pressure drops. Moreover, inlet and outlet forces arising due to flow transitions between the cylinder and the helical tube as well as shear effects were also considered in the complete treatment by Swift et al. (2013).

|  |
| --- |
|  |

Figure 1. Sketch of the helical fluid inerter: front view (left) and cross section (right)

Referring again to the work by Swift et al. (2013) for a detailed derivation of these additional effects, we limit ourselves to analyse how these different contributions affect the total damping force of the device. Shown in Figure 2 is the plot of the total damping force as well as of the different force contributions for the fluid inerter device described in Glover et al. (2009) for a piston velocity in the range [0-1] m/s.

|  |
| --- |
|  |

Figure 2. Different contributions to the total damping force of the fluid inerter described in Glover et al. (2009)

It is easily seen that the main damping contribution is due to the viscous damping effects occurring in the helical tube, which are described by the following expression

 (2)

where  represents a pressure drop, while  and  denote the dynamic viscosity and the mass density of the fluid at reference temperature, respectively. Based on these considerations, the fluid inerter device is idealized as a linear inerter in parallel with a nonlinear damper, with a power law damping term having exponent equal to 1.75 (De Domenico et al. 2018a).

|  |
| --- |
|  |

Figure 3. Experiments on a fluid inerter and force time-history under sinusoidal motion from Smith and Wagg (2016)

The effectiveness of this assumption is verified against experimental results obtained at the Department of Mechanical Engineering and reported in earlier studies (Smith and Wagg 2016). The geometrical parameters of the fluid inerter can be found in (Smith and Wagg 2016). Figure 3 shows the comparison between experimental results (corresponding to a sinusoidal excitation having amplitude 17.5mm and frequency 3Hz) and analytical predictions. The analytical force is computed based on the sum of inertance force (1) and damping force (2) for the given geometrical parameters of the device and physical parameters of the employed fluid (with the addition of a friction force  that was identified by the experiments, with ).

By inspection of Figure 3, it emerges that the forces in the experiment are well captured by the proposed simplified analytical model. However, it is worth noting that the friction force considered here (and added to the damping and inertance contributions) is generally negligible in earthquake engineering applications due to the involved velocities and accelerations that produce forces much higher than the typical friction contributions. Consequently, the idealization of the fluid inerter as an inerter in parallel with a nonlinear dashpot seems to be quite accurate to capture the hysteretic characteristics of this device.

**3. Base isolated structures with TMD and fluid inerter**

The fluid inerter device, whose main characteristics have been summarized in the previous Section, is here proposed for earthquake protection purposes in combination with the seismic base isolation. Figure 4 depicts two alternative installation configurations of the fluid inerter in conjunction with a TMD installed below the base-isolation floor of a building structure. The *n*-story structure is modeled as a lumped-mass system, while the seismic base isolation is represented in terms of the stiffness and viscous damping parameters , respectively (De Domenico et al. 2018c). The TMD is represented by a mass  connected to the isolation floor through an additional set of isolators (called “auxiliary isolators”) having linear stiffness  and linear viscous damping . Finally, the fluid inerter is installed in two alternative configurations as sketched in Figure 4: in the first configuration the device is grounded (left-hand figure) and we call this system “TMDI with parasitic damping” (TMDI-PD), while in the second configuration the fluid inerter is placed in between the TMD mass and the base-isolation floor (right-hand figure), and we call this system “fluid-inerter-based TMD” (FIB-TMD).

|  |
| --- |
|  |

Figure 4. Base isolated structure with TMD and fluid inerter placed according to two different configurations: TMDI with parasitic damping (TMDI-PD) (left), and fluid-inerter-based TMD (FIB-TMD) (right)

The assembled system has thus *N*=*n*+2 degrees of freedom (DOFs), namely the *n* displacements of the base-isolated superstructure , the displacement of the base-isolation floor  and the displacement of the TMD  (equipped with the fluid inerter device).

After introducing the displacement of the superstructure with respect to the base-isolation floor , with  a  vector of ones, the equations of motion of the base-isolated structure equipped with the TMDI-PD system subject to a ground motion acceleration  can be expressed by

 (3)

and those of the base-isolated structure with the FIB-TMD system are given by

 (4)

In the above Eqns. (3) and (4)  represents the force from the first floor of the superstructure to the base-isolation floor;  and  is the force transmitted from the TMD to the base-isolation floor via the set of auxiliary isolators in the two installation configurations (with  denoting the displacement of the TMD with respect to the base-isolation floor);  and  is the (nonlinear) resisting force of the fluid inerter in the two installation configurations (with  denoting the signum function).

On the other hand, we adopt a stochastic description of the ground motion acceleration  to capture the randomness (uncertainty in space, size and time) of the seismic excitation. The widely used two-filter Kanai-Tajimi-Clough-Penzien (Clough and Penzien 2003) power spectral density (PSD) is adopted in this paper, as reported below

 (5)

where the filter parameters  can be calibrated based on the soil conditions (Der Kiureghian and Neuenhofer 1992), while the intensity level  can be related to the peak ground acceleration (PGA) according to the formula  (Buchholdt 1997).

To handle the nonlinear terms of the equations of motion through the random vibration theory, the stochastic linearization technique (SLT) is applied (Robert and Spanos 1990; De Domenico and Ricciardi 2018c, 2019). The SLT allows the replacement of the nonlinear damping force of the fluid inerter device with a linearized term

 (6)

where the linearization damping coefficient  is expressed as follows (De Domenico et al. 2018a)

 (7)

with  denoting the gamma function and  representing the standard deviation of . The latter is related to the sought parameter , therefore an iterative procedure involving input-output relationships in the frequency domain is carried out to linearize the equations.

Once the equations are linearized, the selection of the fluid inerter parameters can be performed based on the minimization of the normalized displacement variance , where  denotes the variance of the base-isolated building without TMD and without fluid inerter and  the corresponding quantity in the improved seismic base isolation. For the optimal design, we assume the following realistic set of coefficients for the PSD, and dynamic parameters of the base-isolation system and TMD:

 (8)

where  is the first-mode damping ratio of the superstructure,  is the corresponding first-mode period,  and  the total mass of the base-isolated structure (including the base-isolation floor) and of the superstructure, respectively,  is the damping ratio of the base-isolation system, and  the corresponding period. Based on these assumed values for the seismic excitation PSD and dynamic properties of the base-isolated building, optimal parameters of the fluid inerter and optimal stiffness and damping ratio of the TMD that minimize the top-floor displacement variance of the base-isolated building are identified as follows

 (9)

***3.1 Sensitivity analysis and variation of fluid inerter properties***

Once the optimal parameters of the fluid inerter are determined as reported in (9), we investigate the sensitivity of the results to variations of the fluid inerter properties (that are intrinsic to the complex experimental behavior described above). There are a number of sources of deviations from the ideal behavior assumed above, including: 1) variability of intensity of the earthquake excitation that triggers a wide range of velocities during the response; 2) transition from laminar to turbulent flow regime during the response, which implies that different analytical formulations would be required to describe the fluid-induced damping effects; 3) imperfect knowledge of the geometrical parameters, or misestimation of physical properties of the fluid, or simply inability of accounting for the evolution of the parameters.

Consequently, it seems interesting to scrutinize, through a numerical sensitivity analysis, how much the performance of the system is negatively influenced by such variations. All the above variations can be represented, in a simplified manner, by corresponding deviations of the inertance and damping values in comparison with the original (optimized) ones. The optimal parameters are , and the corresponding achieved SPI1,min=0.12412.

|  |
| --- |
|  |

Figure 5. Sensitivity analysis for the TMDI-PD system to variations of fluid inerter parameters : a) logarithmic scale in a [0.1-10] range of variation; b) linear scale in a [0.5-1.5] range of variation

We explore a wide range of variations of these parameters within the interval  and , and for each couple of  of deviated parameters we re-compute the SPI1 as a measure of the system performance (degradation). Contour plots of the  ratios are illustrated in Figure 5. Obviously, performance degradation is related to an increased value of the performance ratio , since a unitary value is obtained for . For the wide interval of variation considered, in the logarithmic scale plot of Figure 5a) the  ratio reaches its maximum value (equal to around 4.5) corresponding to the lowest combination of inertance and damping, namely for the couple . Nevertheless, this implies an enormous discrepancy of 90% in the assessment of the fluid inerter parameters, which seems to be very high. Considering the effects of variation mentioned above, it is more reasonable to scrutinize a narrower and more realistic range of  variations of the original parameters. Corresponding results in a linear scale plot are reported in Figure 5b): in this case the maximum  ratio is nearly 1.25 for the lowest couple of parameters, meaning a 25% degradation compared to the optimized performance. Conversely, if the actual inertance and damping parameters are larger than the optimal ones, the performance degradation is negligible (within 5%). Furthermore, it is seen that the inertance is a more sensitive parameter than the damping, as the contour plot levels are narrower along the *x*-axis (meaning that the variations are more pronounced) and broader along the *y*-axis. Consequently, it is expected that the uncertainty in the fluid inerter parameters is not a major issue for the structural control performance of the proposed fluid inerter based TMD systems, provided the modelling assumptions presented in Section 2 are reasonably accurate to capture the physics of the problem.

***3.2 Design of the fluid inerter on the basis of coupled inertance and damping coefficients***

We assume the fluid inerter device is installed in a five-story structure with floor mass  at each story and lateral stiffness  at each story, which leads to a first-mode natural period . A full damping matrix is constructed based on a modal damping matrix such that the modal damping ratio is  for all the vibration modes of the structure. Assuming a basement mass , the structure has an overall mass . Low-damping rubber bearing isolators are assumed, giving a damping ratio  and effective lateral stiffness , which corresponds to a natural period . Considering a mass ratio  and assuming , and stiffness and damping properties of the auxiliary isolators , , respectively, leads to optimal fluid inerter parameters , i.e., the ones already found in the above subsection. These parameters correspond to inertance  and nonlinear damping .

Table 1. Fixed parameters of the fluid inerter design

|  |  |
| --- | --- |
| **Symbol [units]** | **Value** |
| [m] | 0.1 |
| [m] | 0.06 |
| [m] | 1.0 |
| [kg/m3] | 1000 |
| [Pa s] | 0.001 |

We aim to design a fluid inerter device corresponding to such optimal parameters. The goal is to identify the geometry of the device and the physical properties of the fluid that lead to such optimal inertance and damping parameters. However, we emphasize that inertance and damping are coupled characteristics, because changes in one parameter (e.g., the inertance) implies modification of the other parameter (e.g., damping), cf. again Eqns. (1) and (2). Therefore, inertance and damping affect each other and cannot be designed in an independent manner. In reality, although they depend upon many geometrical parameters, it was demonstrated (Deastra et al. 2019) that only two of them are really significant for the design, namely the radius of the piston  and the radius of the helical channel , while the other parameters can be fixed as shown in Table 1.

Fixing all these parameters, Eqns. (1) and (2) become functions of the two unknowns  and  only, which can then be determined through a parametric approach. Assuming the helical pitch  and the number of turns of the helical channel  leads to the results shown in Figure 6, where we draw different design curves by varying  and  in order to identify the couple  corresponding to the optimal values  and . The obtained values of the radii are  and , which corresponds to and . The resulting fluid inerter is sketched in Figure 7, whereby it can be noted that a quite compact device is obtained that is capable of ensuring the sought inertance and damping properties to the improved base isolation system.

|  |
| --- |
| fig14 |

Figure 6. Identification of geometrical properties of the fluid inerter based on parametric approach: a) nonlinear damping versus  for different  curves (left); b) inertance versus  for different  curves (right)

|  |
| --- |
|  |

Figure 7. Sketch of the fluid inerter device based on the optimal design described in this study (units in mm)

**4. Effectiveness of the proposed “improved base-isolation systems”**

The same benchmark five-story base-isolated structure reported in Section 3.2 is considered for assessing the seismic performance of the proposed system. In order to perform a response history analysis with an ensemble of realistic ground motions, 44 seismic inputs extracted from the FEMA P695 (FEMA P695 2009) far field record set have been selected (PEER NGA database, site class C/D). The 44 ground motions, with a PGA ranging from 0.21 to 0.82 [g], have been preliminarily scaled in order to have a PGA = 0.3 [g]. Response history analysis is performed by integrating the equations of motion (3) and (4) through a Runge-Kutta algorithm of fourth order. The seismic performance is evaluated in terms of average maximum (MAX) and root-mean-square (RMS) values of some selected response indicators 

 (10)

|  |
| --- |
| fig10 |

Figure 8. Average RMS and MAX values of response of base-isolated structure with fluid inerter device and TMD compared to conventional base-isolated structure (without TMD) and fixed-base building under 44 ground motions

The profiles of average MAX and RMS values of displacement and absolute acceleration response are comparatively depicted in Figure 8. In particular, we compare: 1) base-isolated building (BI bldg); 2) base-isolated building with TMDI-PD as per the left-hand side of Figure 4; 3) base-isolated building with FIB-TMD as per right part of Figure 4; 4) fixed-base building. The acronym IF in the vertical axis of Figure 8a) and b) stands for “isolation floor”. It is noted that the displacement demand corresponding to the improved seismic base isolation systems (with either TMDI-PD or FIB-TMD) is much lower than the conventional base-isolation system, reduced to approximately 50% (RMS values) and 40% (MAX values). Moreover, the absolute acceleration profile is also drastically reduced by the presence of the fluid inerter device in both the TMDI-PD and FIB-TMD systems, with values around 50% and 45% lower (in terms of RMS and MAX values, respectively). Moreover, that the average response indicators of the system with TMDI-PD are slightly lower than those of the FIB-TMD, which indicates a better seismic performance of the former system.

An example of a time history response is illustrated in Figure 9 for the Imperial Valley ground motion. Shown in Figure 9 are the displacement of the last-floor (fifth story) and the stroke of the auxiliary isolators (of the TMD, ) compared to the stroke of the conventional isolators (of the base-isolation system without TMD, ). It can be noted that the presence of the fluid inerter in combination with the TMD improves the seismic performance of the seismic base isolation significantly. When comparing the two proposed systems, it emerges that the TMDI-PD and the FIB-TMD achieve a comparable seismic performance in terms of displacement demand reduction, but the FIB-TMD is slightly better for reducing the stroke of the isolators.

The main implications of the graphs reported in Figure 8 and Figure 9 are related to the possibility of reducing the displacement demand of the base-isolated structure while simultaneously improving the overall seismic performance of the superstructure. The first aspect has several advantages because it may reduce the risk of pounding in densely populated areas, and it may allow the adoption of smaller isolators for the base isolation system (thus reducing the cost of the base-isolation system). Moreover, the presence of the TMD in the improved base-isolation system is beneficial for reducing the vulnerability to long-period ground motions (Ariga et al. 2006; Takewaki et al. 2011).

Although limited to a quite simple example, the proposed “improved base-isolation systems” seems to have great potential for earthquake engineering applications. The inherent damping properties and the mass-amplification effects induced by the (ideal) fluid inerter device are beneficial for improving the performance of base-isolated structures, by reducing the displacement demand of the isolators and the acceleration response of the superstructure.

|  |
| --- |
| fig12 |

Figure 9. Example of time history of response of proposed systems compared to conventional base isolation (without TMD and without fluid inerter) for the Imperial Valley 1979 earthquake

**5. Conclusions**

This paper has focused on the fluid inerter, a novel device (patented quite recently) that up to now has been mainly used for mechanical engineering applications. Instead, the paper has focused on how to use this novel device in the field of earthquake engineering, particularly, for developing improved seismic base isolation schemes in combination with a TMD system. There are two motivations for using this device in such civil engineering applications: 1) as an inerter device, it provides the system with beneficial mass-amplification effects by modifying the inertial properties and acting as an additional, apparent mass, which is really useful for improving the performance of TMD systems without requiring large physical masses; 2) the fluid variant of the device, in comparison with the rack-and-pinion inerter and the ball-screw inerter (Ikago et al. 2012; Mirza Hessabi and Mercan 2016), offers certain advantages because it avoids backlash and friction phenomena typical of mechanical counterparts, and also provides inherent damping due to pressure drops occurring in helical channel, related to the viscosity of the internal fluid. Based on some experimental observations of a fluid inerter prototype, the device has been modeled as an inerter in parallel with a nonlinear damping element with a power-law nonlinearity having exponent equal to 1.75.

The study has presented two possible installation schemes of the fluid inerter below the base-isolation floor of base-isolated buildings. In both the installation configurations, it has been demonstrated that the presence of the fluid inerter dramatically improves the performance of the attached mass (TMD), and significantly reduces the displacement demand of the isolators, as well as the acceleration response of the superstructure. These conclusions have been drawn based on response history analysis of a simple five-story base-isolated structure modeled under a suite of 44 real accelerograms extracted from the FEMA P695 far-field record set, and evaluating the average RMS and MAX values of selected response indicators. In particular, displacement demand and absolute acceleration are reduced of around 40-45% in terms of MAX values, and approximately 50% in terms of RMS values.

While the simplified model of inerter and nonlinear damping in parallel captures the physics of the problem, it is expected that variations of device parameters occur during a real seismic event. To scrutinize these effects, the sensitivity of the results to variations of the device parameters from the optimal ones has been discussed: variations in the range of  of inertance and damping coefficients lead to approximately a 25% degradation compared to the optimized performance.

In the authors’ opinion, the fluid inerter is a suitable candidate for being employed in earthquake engineering applications, with much more interesting aspects than the widely used fluid viscous dampers. Laboratory experiments are certainly necessary to analyze the mechanical behaviour of the fluid inerter device (e.g., the prototype designed in this paper and depicted in Figure 7) under a more realistic range of forces and velocities typically involved in earthquake engineering applications. Additionally, a scaled base-isolated frame model with implementation of the proposed “improved base-isolation systems” would provide experimental validation of the seismic performance, which represents the object of ongoing research.

**6. Acknowledgments**

The authors would like to acknowledge the contribution by N.D. Smith and technical staff from the Department of Mechanical Engineering, University of Sheffield for the experimental testing activities described in Section 2 of this paper. The second author PD acknowledges the financial support from the Indonesia Endowment Fund F or Education (LPDP).

**7. References**

Ariga T., Kanno Y., Takewaki I. (2006) Resonant behaviour of base-isolated high-rise buildings under long-period ground motions. Structural Design of Tall and Special Buildings 15: 325–338.

Buchholdt H. (1997) *Structural dynamics for engineering*. London: Thomas Teldfort.

Clough R.W., Penzien J. (2003) *Dynamics of Structures*, 3rd ed. Berkeley, CA, USA: Computers and Structures Inc.

De Domenico D., Deastra P., Ricciardi G., Sims N.D., Wagg, D.J. (2018a). Novel fluid inerter based tuned mass dampers for optimised structural control of base-isolated buildings. *Journal of the Franklin Institute*, DOI: 10.1016/j.jfranklin.2018.11.012.

De Domenico D., Impollonia N., Ricciardi G. (2018b). Soil-dependent optimum design of a new passive vibration control system combining seismic base isolation with tuned inerter damper. *Soil Dyn Earth Eng*, 105: 37-53.

De Domenico D., Falsone G., Ricciardi G. (2018c) Improved response-spectrum analysis of base-isolated buildings: a substructure-based response spectrum method. Eng Struct 162: 198-212.

De Domenico D., Ricciardi G. (2018a) An enhanced base isolation system equipped with optimal Tuned Mass Damper Inerter (TMDI). *Earthq Eng Struct Dyn*, 47: 1169-1192.

De Domenico D., Ricciardi G. (2018b) Optimal design and seismic performance of tuned mass damper inerter (TMDI) for structures with nonlinear base isolation systems. *Earthq Eng Struct Dyn*, 47: 2539-2560.

De Domenico D., Ricciardi G. (2018c) Improved stochastic linearization technique for structures with nonlinear viscous dampers. *Soil Dyn Earth Eng*; 113: 415-419.

De Domenico D., Ricciardi G. (2018d). Earthquake-resilient design of base isolated buildings with TMD at basement: application to a case study. *Soil Dyn Earth Eng*, 113: 503-521.

De Domenico D., Ricciardi G. (2018e). Improving the dynamic performance of base-isolated structures via tuned mass damper and inerter devices: a comparative study. *Struct Control Health Monit*, 25(10): e2234.

De Domenico D., Ricciardi G. (2019). Earthquake protection of structures with nonlinear viscous dampers optimized through an energy-based stochastic approach. *Eng Struct*, 179: 523-539.

De Domenico D., Ricciardi G., Takewaki I. (2019). Design strategies of viscous dampers for seismic protection of building structures: A review. *Soil Dyn Earth Eng*, 118: 144-165.

Deastra P., Wagg D.J., Sims N.D. (2019) The realisation of an inerter-based system using fluid inerter. In Dynamics of Civil Structures, Volume 2 (pp. 127-134). Springer, Cham.

Der Kiureghian A., Neuenhofer A. (1992) Response spectrum method for multi-support seismic excitations. *Earth Eng Struct Dyn*, 21: 713-740.

FEMA P695 (Federal Emergency Management Agency) (2009) Quantification of building seismic performance factors. Federal Emergency Management Agency, Washington, D.C.; 2009.

Glover A.R., Smith M.C., Houghton N.E., Long P.J.G. (2009) Force-controlling hydraulic device (International Patent Application No: PCT/GB2010/001491, 2009).

Gonzalez-Buelga A., Lazar I., Jiang J.Z., Neild S.A., Inman D.J. (2017) Assessing the effect of nonlinearities on the performance of a tuned inerter damper. *Struct. Control Health Monit.*, 24(3): e1879.

Ikago K., Saito K., Inoue N. (2012) Seismic control of single‐degree‐of‐freedom structure using tuned viscous mass damper, *Earthq Eng Struct Dyn* 41(3): 453-474.

Lazar I.F., Neild S.A., Wagg D.J. (2014) Using an inerter-based device for structural vibration suppression. *Earthq Eng Struct Dyn* 43: 1129-1147.

Marian L., Giaralis A. (2014) Optimal design of a novel tuned mass-damper-inerter (TMDI) passive vibration control configuration for stochastically support-excited structural systems. *Prob Eng Mech* 38: 156-164.

Mirza Hessabi R., Mercan O. (2016) Investigations of the application of gyro-mass dampers with various types of supplemental dampers for vibration control of building structures. *Eng Struct* 126: 174-186.

Roberts J.B., Spanos P.D. (1990) *Random Vibration and Statistical Linearization*. New York: Wiley.

Smith M.C. (2002) Synthesis of mechanical networks: the inerter. *IEEE Transactions on Automatic Control*, 47(10): 1648-1662.

Smith N., Wagg D. (2016) A fluid inerter with variable inertance properties. In: *6th European Conference on Structural Control (EACS2016)*, Sheffield, UK, 11-13 July 2016, paper no. 199.

Swift S.J., Smith M.C., Glover A.R., Papageorgiou C., Gartner B., Houghton N.E. (2013) Design and modelling of a fluid inerter. *Int J Control*, 86(11): 2035-2051.

Takewaki I., Murakami S., Fujita K., Yoshitomi S., Tsuji M. (2011) The 2011 off the Pacific coast of Tohoku earthquake and response of high-rise buildings under long-period ground motions. *Soil Dyn Earth Eng* 31(11): 1511–1528

Taniguchi T., Der Kiureghian A., Melkumyan M. (2008) Effect of tuned mass damper on displacement demand of base-isolated structures. *Eng Struct*, 30: 3478-3488.

1. PhD – post-doc researcher, Dept. of Engineering, University of Messina, Messina, Italy, [dario.dedomenico@unime.it](mailto:dario.dedomenico@unime.it) [↑](#footnote-ref-1)
2. PhD candidate, Dept. of Mechanical Engineering, University of Sheffield, Sheffield, UK, [pdeastra1@sheffield.ac.uk](mailto:pdeastra1@sheffield.ac.uk) [↑](#footnote-ref-2)
3. PhD – professor, Dept. of Engineering, University of Messina, Messina, Italy, [gricciardi@unime.it](mailto:gricciardi@unime.it) [↑](#footnote-ref-3)
4. PhD – professor, Dept. of Mechanical Engineering, University of Sheffield, Sheffield, UK, [n.sims@sheffield.ac.uk](mailto:n.sims@sheffield.ac.uk) [↑](#footnote-ref-4)
5. PhD – professor, Dept. of Mechanical Engineering, University of Sheffield, UK, [david.wagg@sheffield.ac.uk](mailto:david.wagg@sheffield.ac.uk) [↑](#footnote-ref-5)