**DEFORMATION-HISTORY INTEGRAL TYPE HYSTERESIS MODEL CONSIDERING PERFORMANCE CHANGE FOR HIGH-DAMPING RUBBER BEARINGS**

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**ABSTRACT**

In the case of time history analysis to design seismically isolated structures, analytical modeling of isolated layer is important. There are many types of isolator device, however, all devices show performance change more or less. For example, production variation, temperature dependence, aging, repeated loading dependence, compressive stress dependence and so on. Therefore, it is important to select proper hysteresis model which can represent the performance change accurately. In the past, authors proposed new hysteresis model for high-damping rubber bearings (HDR) which can represent complex behavior of HDR. The model is called Deformation-History Integral type model (DHI model). However, it was not clear how to represent the performance change. In this study, method of representing the performance change for DHI model is proposed.

*Keywords: Seismically isolated structure; High-damping rubber bearings; Hysteresis model; Time history analysis; Performance change*

**1. INTRODUCTION**

Seismic isolation has gained popularity as one of countermeasures for seismic protection of structures in these decades (Murota 2009). The seismic isolation is an aseismic design concept to reduce the seismic force transmitted to the structure by supporting it with a flexible element – elastomeric isolators – at the base or sometimes middle story of the buildings, to elongate the natural period of the structure and thereby decouples it from the ground. Basically, seismic isolation systems provide functions of restoring force and energy dissipation.

Laminated rubber bearing (RB) is one of the seismic isolation devices. In Japan, the elastomeric isolator, made up with layers of alternating rubber and steel plates is the most popular device for providing restoring force and damping characteristics.

Generally, in the case of time history analysis to design seismically isolated structures, analytical modeling of isolated layer is important. There are many types of isolator device, however, all devices show performance change more or less. For example, production variation, temperature dependence, aging, repeated loading dependence, compressive stress dependence and so on. Therefore, it is important to select proper hysteresis model which can represent the performance change accurately. In the past, authors proposed new hysteresis model for high-damping rubber bearings (HDR) which can represent complex behavior of HDR. The model is called Deformation-History Integral type model (DHI model) (Kato et al. 2014). However, it was not clear how to represent the performance change so far. In this study, therefore, method of representing the performance change for DHI model is proposed.

**2. OUTLINE OF ORIGINAL DHI MODEL**

In this chapter, outline of original DHI model is introduced.

***2.1 Outline of DHI model***

Bi-directional DHI model is defined by relation between shear stress and shear strain as follows;

(1)

(2)

(3)

First and second term in Equations (1) - (2) represent elasticity with damage effect and plasticity, respectively. N is number of plastic branches. *γ*1 and *γ*2 are shear strain for *x*1 and *x*2 direction, respectively. *τ*1 and *τ*2 are shear stress for *x*1 and *x*2 direction, respectively. *a*, *b*, *c*1, … *c*N, *l*1, … and *l*N are material parameters. As can be seen from Equation (1) - (3), DHI model is defined by line integral along deformation orbit *χ* of *γ*1 - *γ*2 plane under bi-directional pure shear deformation (see Figure 1). Example of orbit *χ* of *γ*1 - *γ*2 plane is shown in Figure 2. *Ξ* is damage function and is defined as;

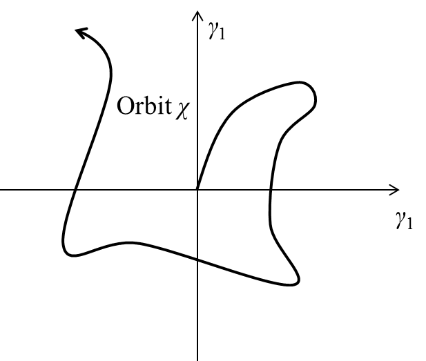
 (4)

 (5)

Where, *θ* and *β* are material parameters. *γ*m is maximum shear strain experienced in the past.



Figure 1. Bi-directional pure shear deformation.



*γ*1

*γ*2

Figure 2. Example of deformation orbit *χ*.

***2.2 Feature of DHI model***

DHI model is plastic-elastic model and not depends on loading velocity or frequency. DHI model seems complex model at first glance, however, there are many merits of using DHI model as follows;

1. DHI model can represent the complex behavior of HDR including bi-directional loading for wide range of shear strain.

2. It is very easy to implement DHI model in structural analysis software, because calculation algorithm is very simple. In other words, there is no need to use if statement, branch and iteration in calculation algorithm.

3. DHI model was originally developed for finite element analysis (FEA) (T. Mori et al. 2010). DHI model for FEA (hereinafter referred to as DHI-FEA model) is defined as stress tensor – strain tensor relationship and the DHI-FEA model has 6 degrees of freedom. Bi-directional DHI model defined by equation (1) - (3) is derived by decreasing 4 degrees of freedom of DHI-FEA model. Therefore, same material parameter values can be used for FEA model and time history analysis model.

4. DHI model can represent the complex behavior of HDR by just only 3 parameters. In this paper, however, 8 parameters are used in order to represent behavior of HDR more accurately.

5. While DHI model is not viscoelastic but elastic-plastic model, DHI model can represent the creep behavior of HDR. Regarding HDR, creep behavior is critical under wind response because wind force is loaded for one direction. Therefore, DHI model can represent the behavior under wind force loading such as typhoon as well as earthquake.

As for more details, refer to original paper on DHI model (Kato et al. 2014).

**3. PROPOSED DHI MODEL CONSIDERING PERFORMANCE CHANGE**

In this Chapter, method of representing performance change for DHI model is proposed.

***3.1 DHI model considering performance change***

Outline of DHI model considering performance change is as follows.

Performance change ratio *x* and *y* are defined as following equation. *x* and *y* are performance change ratio for *G*eq and *H*eq, respectively.

(6)

(7)

*G*eq,c and *H*eq,c are shear stiffness and equivalent damping ratio of performance changed state, respectively. *G*eq,0 and *H*eq,0 are shear stiffness and equivalent damping ratio of standard state, respectively. In Japan, for example, temperature and loading frequency of standard state are 20 degree Celsius and 0.33Hz (sinusoidal), respectively. Here, parameters considering performance change *a’*, *b’*, *c*1’, … *c*N’, *l*1’, … *l*N’, *θ’* and *β’* are defined. By defining these parameters as function of *x*, *y* and parameters of standard state *a*, *b*, *c*1, … *c*N, *l*1, … *l*N, *θ* and *β*, DHI model considering performance change can be clarified. The conversion functions are shown in Table.1. *A* is an adjustment parameter and is determined by minimizing the error shown in Table 3 next section. Value of *A* is differs for each rubber material. The validity of the conversion function in Table 1 will be confirmed in next section 3.2.

Table 1. Conversion functions from standard state to performance changed state.

|  |  |
| --- | --- |
| **Parameters** | **Conversion formula** |
| *θ*’, *β*’, *l*n’ | *θ*, *β*, *l*n |
| *c*n’ | *c*n’= *xy*(1+*A*-*Ay*)(1-*A*+*Ay*)*c*n |
| *a’* |  |
| *b’* |  |

***3.2 Validation of proposed DHI model***

Examples of shear stress – shear strain relationship for various values of *x* and *y* are shown in Figure 3. Example of HDR is called X0.6R which has 0.620 MPa of shear modulus and 0.240 of equivalent damping ratio (Bridgestone 2018). Material parameters of standard state are shown in Table 2. As can be seen from Table 2, number of branches for X0.6R is 2. Material parameter value *A* for X0.6R is 0.020209. Comparison of error between *x*, *y* and performance change ratio calculated from hysteresis loop of DHI model is shown in Table 5. As can be seen from Table 3, error is less than 0.1% for applicable scope of *x* and *y* (applicable scope is 0.4≤*x*, *y*≤1.6). Thus validation of proposed model is confirmed.

In Figure 4, comparison of shear strain dependence between proposed DHI model considering performance change and design equation. *G*eq and *H*eq are normalized by their values at shear strain *γ*=1.0. As an example, 2 cases of *x*=*y*=0.8 and *x*=*y*=1.2 are shown. Design equation is determined by shear dependence test using full-scale and scaled model. Generally, seismically isolated structure is designed under the assumption shear strain dependency does not depend on performance change. (Some models are considering performance dependence of shear strain dependence, e.g. Kikuch-Aiken model considers the compressive stress dependence (Kikuch 2010).) Number of example cases is only 2, however, as can be seen from Figure 4, shear dependence calculated by proposed DHI model shows good agreement with design equation independent of *x* and *y* values.

Table 2. Parameter values under standard state of X0.6R.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ***θ*(-)** | ***β*(-)** | ***a* (MPa)** | ***b* (MPa)** | ***c*1 (MPa)** | ***l*1 (-)** | ***c*2 (MPa)** | ***l*2 (-)** |
| 0.3696 | 0.4578 | 0.7189 | 0.02796 | 3.237 | 0.03006 | 0.5128 | 0.3592 |

Table 3. Error between *x*, *y* and performance change ratio calculated from hysteresis loop.

|  |  |  |  |
| --- | --- | --- | --- |
| ***x*** | ***y*** | **Error of *G*eq** | **Error of *H*eq** |
| Any value of *x*>0 | 0.4 | Less than 0.1% | Less than 0.1% |
| 0.7 |
| 1.0 |
| 1.3 |
| 1.6 |

|  |  |  |
| --- | --- | --- |
|  |  |  |
| (a) *x*=0.8, *y*=0.8 | (b) *x*=1.0, *y*=0.8 | (c) *x*=1.2, *y*=0.8 |
|  |  |  |
| (d) *x*=0.8, *y*=1.0 | (e) *x*=1.0, *y*=1.0 (Standard state) | (f) *x*=1.2, *y*=1.0 |
|  |  |  |
| (g) *x*=0.8, *y*=1.2 | (h) *x*=1.0, *y*=1.2 | (i) *x*=1.2, *y*=1.2 |

Figure 3. Examples of shear stress – shear strain relationship considering performance change

|  |  |
| --- | --- |
|  |  |
|
| (a) *G*eq (*x*=0.8, *y*=0.8) | (b) *H*eq (*x*=0.8, *y*=0.8) |
|  |  |
| (c) *G*eq (*x*=1.2, *y*=1.2) | (d) *H*eq (*x*=1.2, *y*=1.2) |

Figure 4. Comparison of shear strain dependence between design equation  
 and proposed DHI model for some cases of *x*, *y*.

**4. EXAMPLE OF TIME HISTORY ANALYSIS**

In this chapter, examples of time history analysis result using proposed DHI model with performance change are shown.

***4.1 Outline of time history analysis model***

Analysis model and chosen building model in this paper are shown in Figure 5. In Figure 5, layout of seismically isolated layer is shown as well. Building model is 42-story high-rise building and analysis model is uni-directional or bi-directional spring-mass model (number of masses is 43). All isolators are X0.6R type high-damping rubber bearings shown in previous Chapter 3. Seismically isolated layer is modeled by only DHI model and upper structure is modeled by linear-spring with viscous damping. Analysis cases and input seismic waves are shown in Table 4 and Figure 6, respectively. For each analysis case in Table 4, 3 states of analysis were carried out, i.e. the lowest shear stiffness state (hereinafter referred to as Lower bound), standard state (hereinafter referred to as Standard) and the highest shear stiffness state (hereinafter referred to as Upper bound). Therefore total number of analysis case is 9. *x*, *y* values and its parameter values are shown in Table 5 and 6, respectively. Performance change factors of HDR are temperature dependence, aging and production variation. Temperature of Lower and Upper bound are 40 and 0 degree Celsius, respectively. Aging of Lower and Upper bound are 60 and 0 years, respectively. Production variation of Lower and Upper bound are +10% and -10%, respectively. As for more details regarding performance change, refer to technical report on X0.6R (Bridgestone 2018).

Table 4. Analysis cases.

|  |  |  |
| --- | --- | --- |
| **Analysis case** | **Horizontal direction** | **Input wave** |
| El Centro | Uni-direction | El Centro NS |
| JMA Kobe Uni-direction | Uni-direction | JMA Kobe NS |
| JMA Kobe Bi-direction | Bi-direction | JMA Kobe NS  JMA Kobe EW |

Table 5. *x* and *y* values for each state.

|  |  |  |
| --- | --- | --- |
| **State** | ***x*** | ***y*** |
| Lower bound | 0.87 | 0.97 |
| Standard | 1.00 | 1.00 |
| Upper bound | 1.41 | 0.74 |

Table 6. Parameter values of upper bound, standard state and lower bound of X0.6R.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | ***θ*(-)** | ***β*(-)** | ***a* (MPa)** | ***b* (MPa)** | ***c*1 (MPa)** | ***l*1 (-)** | ***c*2 (MPa)** | ***l*2 (-)** |
| Lower | 0.3696 | 0.4578 | 1.1446 | 0.04452 | 3.971 | 0.03006 | 0.6290 | 0.3592 |
| Standard | 0.7189 | 0.02796 | 3.237 | 0.5128 |
| Upper | 0.5478 | 0.02131 | 2.324 | 0.3681 |

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 1. Analysis model | 1. Building model | 1. Layout of seismically isolated layer |

Figure 5. Analysis model, chosen building model in this paper and layout of seismically isolated layer.

|  |  |
| --- | --- |
|  |  |
| 1. El Centro NS | 1. JMA Kobe NS |
|  |  |
| (c) JMA Kobe EW |  |

Figure 6. Input seismic wave.

***4.2 Analysis results***

Floor response results are shown in Figure 7, 8 and 9. Hysteresis loop results of seismically isolated layer are shown in Figure 10. As can be seen from Figure 7 and 10, maximum displacement of isolated layer tend to be larger as the stiffness of isolated layer is smaller. As can be seen from Figure 8 and 9, on the other hand, maximum acceleration and shear coefficient tend to be larger as the stiffness of isolated layer is larger. Obtained results above are natural consequence, however, the usefulness of using DHI model with performance change was confirmed.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1. El Centro | 1. JMA Kobe Uni-direction | NS direction | EW direction |
| 1. JMA Kobe Bi-direction | |

Figure 7. Floor response results (maximum displacement).

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1. El Centro | 1. JMA Kobe Uni-direction | NS direction | EW direction |
| 1. JMA Kobe Bi-direction | |

Figure 8. Floor response results (maximum acceleration).

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1. El Centro | 1. JMA Kobe Uni-direction | NS direction | EW direction |
| 1. JMA Kobe Bi-direction | |

Figure 9. Floor response results (maximum shear coefficient).

|  |  |
| --- | --- |
|  |  |
| 1. El Centro | 1. JMA Kobe Uni-direction |
| Figure 10. Hysteresis loop results of seismically isolated layer. | |
|  |  |
| NS direction | EW direction |
| 1. JMA Kobe Bi-direction | |

Figure 10. Hysteresis loop results of seismically isolated layer (Cont.).

**4. conclusions**

In the past, authors proposed new hysteresis model for high-damping rubber bearings. As the model was not clear how to represent the performance change, the method of representing the performance change of shear stiffness *G*eq and equivalent damping ratio *H*eq is proposed in this paper. Proposed model was verified by comparing performance change ratio with property change ratio calculated analytically from hysteresis loop. In addition, example of time history analysis was shown.

**6. References**

Murota N., Earthquake Protection Materials – Reviews and Future directions of Elastomeric Isolators. (In Japanese), *Polymers* Vol.58, No.6, The Society of Polymer Science, 2009, Japan.

Kato H., Mori T., Murota N., Kikuchi M., Analytical Model for Elastoplastic and Creep-Like Behavior of High-Damping Rubber Bearings, *Journal of Structural Engineering*, Vol. 141, No 9, 2014 , American Society of Civil Engineering.

Mori T., Kato H. and Murota N., FEM Analysis of High Damping Laminated Rubber Bearings Using an Elastic-Plastic Constitutive Law of the Deformation History Integral Type. (In Japanese), *Journal of Structural and Construction Engineering*, Vol.75 No.658, 2171-2178, 2010, Architectural Institute of Japan, Japan.

Bridgestone Co., Technical report (HDR-X0.6R), 2018

Kikuchi, M., Nakamura, T., Aiken, I. D., Three-dimensional analysis for square seismic isolation bearings under large shear deformations and high axial loads., *Earthquake Eng. Struct. Dyn*., 39(13), 1513–1531, 2010.

Computers and Structures Inc., Technical Note High-Damping Rubber Isolator Link Property, 2017.

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