**Top-Story Mass Dampers for Seismic Control of**

**Inelastic Asymmetric-Plan BuildingS**

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**ABSTRACT**

Self-mass dampers, which use intrinsic parts of structures as tuned mass dampers, are economically advantageous in terms of the materials and space required. This study proposes using the top story of a two-way asymmetric-plan building as a self-mass damper, referred to as a top-story mass damper (TSMD), for suppressing the vibrations of the first triplet of vibration modes of the building. The first triplet of vibration modes are the first translational dominant mode in each of the two horizontal directions and the first rotational dominant mode in the vertical direction. Furthermore, this study employs a pair of elastic TSMDs to alleviate the detuning effects caused by yielding of the main structure. One TSMD of the pair is responsible for suppressing the vibrations of the target building in elastic states. The other TSMD, which is designed based on the properties of collective force–deformation relationships, is responsible for suppressing the vibrations of the target building in inelastic states. The collective force–deformation relationships are the pushover curves of the target building when subject to the collective modal inertia force vectors of the first triplet of vibration modes of the building. Numerical examples are used to examine the seismic effectiveness of TSMDs.

*Keywords: Tuned mass damper; Asymmetric-plan building; Self-mass damper; Seismic control; Inelastic seismic response*

**1. INTRODUCTION**

Tuned mass dampers (TMDs) have been widely recognized as an effective approach to reducing displacement demands of elastic buildings that are subject to earthquake loads. Nevertheless, this advantage generally diminishes as detuning progresses. Conventional TMDs, which need no additional power source or sophisticated sensors/instruments, can be regularly and elaborately adjusted to address detuning caused by the alteration of usage or aging of materials. For instance, removing/adding a part of the TMD mass blocks or changing the length of the ropes suspending them can adjust the TMD’s dynamic properties. However, adjusting conventional TMDs to address detuning caused by structural yielding during ground motions is not easily achievable. As a result, the effectiveness of a TMD on reducing displacement demands generally decreases as seismic damage to the main structure increases. It would be desirable for conventional TMDs to also effectively mitigate the displacement demands of inelastic buildings.

As for using TMDs to reduce the displacement demands of elastic asymmetric-plan buildings, a single translation-only TMD is obviously insufficient for addressing the translation-rotation coupled vibrations of asymmetric-plan buildings. The common approach for dealing with this issue is to employ multiple TMDs (MTMDs) (Jangid and Datta 1997, Li and Qu 2006). Recently, Lin (2017) further proposed using the top story of a two-way asymmetric-plan building as a TMD, referred to as a top-story mass damper (TSMD), for the simultaneous control of the first triplet of building vibration modes. The first triplet of vibration modes of a two-way asymmetric-plan building is composed of the first translational dominant mode in each of the two horizontal directions and the first rotational dominant mode in the vertical direction. The self-weight of the top story is designed to provide the needed mass and mass moment of inertia for the TSMD. The required damping and stiffness matrices of the TSMD are achieved by installing viscous dampers and springs/bearings between the bottom of the top story and the top of the stories below. Thus, additional heavy mass blocks are not needed, nor is a permanently occupied huge space for the installation and operation of a mass damper. The economic benefits of the building are thereby improved because of the increased available space. In addition, when remodeling the top story or adding a purposely designed story atop an existing building as a TSMD, the occupancy of the other stories need not to be suspended during the retrofit period. Therefore, TSMDs are very suitable for the seismic retrofit of existing two-way asymmetric-plan buildings. Figure 1 (Lin 2017) illustrates the concept of constructing a TSMD.

Although the seismic effectiveness of a TSMD on an elastic two-way asymmetric-plan building has been validated (Lin 2017), it is not clear whether the TSMD is beneficial or at least harmless for buildings that undergo inelastic excursions in rare/very rare earthquakes. Furthermore, it would be a significant advancement if the generally unremarkable effect of TMDs on mitigating displacement demands of inelastic buildings is improved. Therefore, this study has two goals: the first is to investigate the seismic effectiveness of TSMDs for inelastic two-way asymmetric-plan buildings with different extents of yielding that range from minor to severe damage; the second is to improve the effectiveness of TSMDs in reducing the displacement demands of inelastic asymmetric-plan buildings. Analogous to the idea of employing MTMDs to control a wide band of vibration frequencies (Lin et al. 2017), this study proposes an approach to design a pair of elastic TSMDs to counter the detuning caused by the building deforming from an elastic state to a damaged state.



Figure 1. Concept sketch for constructing a TSMD

**2. A pair of elastic TSMDs**

To mitigate the detuning effect that results from the yielding of two-way asymmetric-plan buildings, a pair of elastic TSMDs, individually denoted as TSMD1 and TSMD2, are designed based on the bilinear force–deformation relationships of the first triplet of vibration modes of the target building. TSMD1, which is responsible for suppressing the vibrations of the target building in elastic states and unloading states, is directly obtained from the design method proposed previously (Lin 2017). TSMD2 is responsible for suppressing the vibrations of the target building in inelastic (i.e., softened) states. The method for determining the bilinear force–deformation relationships of the first triplet of vibration modes of the target building are stated below.

The equation of motion of an *N*-story, two-way asymmetric-plan building with each floor represented as a rigid diaphragm with three degrees of freedom (DOFs) is expressed as follows (Lin and Tsai 2008):



(1)

Here, andare the x- and z-directional ground accelerations, respectively; **ι**x and **ι**z are the x- and z-directional influence vectors, respectively. The displacement vector **u**, the *n*th mode shape, the mass matrix **M**, the damping matrix **C**, the stiffness matrix **K**, and the *n*th modal participation factors *xn*and *zn* are given as follows:



(2)

**s***xn*and **s***zn*, which, respectively, represent the x- and z-directional *n*th modal inertia force vectors, are

(3)



The TSMD properties are obtained from the optimization of the effective one-story building (EOSB), which retains the dynamic properties of the first triplet of vibration modes of the target building (Lin 2017). This indicates that the dynamic response of the EOSB reflect the collective dynamic response of the first triplet of vibration modes of the target building in the modal space. This study further proposes the collective modal inertia force vectors of the first triplet of vibration modes as:



(4)

In which,and. Note that the subscript, *i* = 1 to 3, used in Equation 4, denotes the parameters belonging to the three modes constituting the first triplet of vibration modes, which are not necessarily the first three vibration modes of the target building. It is worth noting that according to the assumption of the modal pushover analysis (MPA) approach (Chopra and Goel 2002), applying the *n*th modal inertia force to a structure triggers only the *n*th modal deformation of the structure, even though the structure is underwent inelastic excursions. Likewise, assume that applying the collective modal inertia force of the first triplet of vibration modes to the target building triggers only the collectivemodal deformations of its first triplet of vibration modes. By subjecting the target building to **s***x*, the relationship between the x-directional roof translation, *ux,r*, and the x-directional base shear, *Vbx*, is obtained. By subjecting the target building to **s***z*, the relationship between the z-directional roof translation, *uz,r*, and the z-directional base shear, *Vbz*, is obtained. In addition, by subjecting the target building to **s****, the relationship between the y-directional roof rotation, *u,r*, and the base torque, *Tb*, is obtained. Note that in the pushover analysis procedure (ATC 1996), the roof displacement and base shear are divided by and, respectively, to convert the force–deformation relationship into the format of acceleration-displacement response spectrum (ADRS). **1,*r* represents the roof component of the first mode shape. In addition, 1 represents the modal participation factor of the first vibration mode and. Similarly, the aforementioned force–deformation relationships, i.e., *Vbx* vs. *ux,r*, *Vbz* vs. *uz,r*, and *Tb* vs. *u,r*, are mapped into the subspace spanned by the first triplet of vibration modes as follows:



(5)

where . Again, the subscripts 1, 2, and 3 used in Equation 5 denote the parameters belonging to the first triplet of vibration modes, which are in sequence from low to high, rather than the first three vibration modes of the target building. All three vibration modes are simultaneously triggered by applying each of **s***x*, **s***z*, and **s**** onto the building. Therefore, the total mass of the three vibration modes should be taken into account (Equation 5). This is why the sum of the effective modal participation mass of the first triplet of vibration modes in the x- and z-directions, and the sum of the effective modal participation mass moment of inertia of the first triplet of vibration modesare used in Equation 5. For the purpose of clarity, **s***x*, **s***z*, and **s**** (quation 4) of an EOSB for which *N* = 1 are specifically denoted as, , and, respectively. Because the first triplet of vibration modes of an EOSB exactly constitutes all of its vibration modes, , , and are just equal to ,, and, respectively, where ,, andare the x-directional mass, z-directional mass, and the mass moment of inertia of the EOSB. In addition, Equation 5 for the case of the EOSB becomes



(6)

The superscript \* used in Equation 6 represents the quantities belonging to the EOSB in order to differentiate these notations from the commonly used notations for the original building.

The collective force–deformation relationships of the EOSB presented in ADRS format (Equation 6) should be the same as those of the target building (Equation 5). In other words, the collective force–deformation relationships of the EOSB can be found by using **s***x*, **s***z*, and **s****to push the target building. The stated force–deformation relationships are then simplified as bilinear curves. The post-yielding stiffness ratios of these three bilinear curves are denoted as *x*, *z*, and **. The elastic segments of the bilinear curves are the collective force–deformation relationships of EOSB1, which was proposed in Lin (2017). EOSB1 is then optimized as TSMD1. TSMD1 is expected to suppress the vibrations of the first triplet of vibration modes in elastic and unloading states. The inelastic segments of the bilinear curves are regarded as the elastic collective force–deformation relationships of EOSB2, which is then optimized as TSMD2. TSMD2 is expected to suppress the vibrations of the first triplet of vibration modes in inelastic states. The schematic drawing of the concept of constructing TSMD1 and TSMD2 is shown in Figure 2. Note that TSMD1 and TSMD2 are activated all the time to suppress the vibrations of the target two-way asymmetric-plan building. Just like the concept of employing MTMDs to control a wide band of vibration frequencies, this study proposes using a pair of elastic TSMDs, i.e., TSMD1 and TSMD2, to counter the detuning caused by the yielding of the target two-way asymmetric-plan building subjected to strong ground motions.



Figure 2. The concept sketch for constructing a pair of elastic TSMDs

For the purpose of simplicity, this study constructed the analytical model of an EOSB as a simple spring-dashpot system (Figure 3), which consists of a lumped mass possessing mass , and mass moment of inertia, three springs, and three linear dashpots. The stiffness values of the three springs are denoted as,, and(Figure 3). In addition, the damping coefficients of the three linear dashpots are denoted as,, and(Figure 3). There are two rigid bars, one of which connects the lumped mass and the three springs and the other connects the lumped mass and the three dashpots. The lengths and directions of these two rigid bars, which are described in terms of ,,, and(Figure 3), are associated with the stiffness eccentricity and the damping eccentricity of the EOSB itself. Because the mass, damping, and stiffness matrices of EOSB1 can be taken from the equations shown in Lin (2017), the aforementioned parameters used for constructing the analytical model of EOSB1 are computed as below:



(7)

In which the notations shown on the right-hand side of Equation 7 are defined by Lin (2017). It is worth remembering that the EOSB with bilinear collective force–deformation relationships, which are obtained by using **s***x*, **s***z*, and **s**** to push the target building, is split into EOSB1 and EOSB2. EOSB1 and EOSB2 represent the same triplet of vibration modes of the target building in the elastic and inelastic states, respectively. Therefore, the analytical model of EOSB2 is the same as that of EOSB1 except for the three spring stiffness values, which are affected by the softening of the triplet of vibration modes. In addition, because the inherent damping of the target building is assumed to be Rayleigh damping, the damping matrix is dependent on the mass and stiffness matrices. Therefore, the three damping coefficients of the analytical model of EOSB2 are also different from those of EOSB1. The three spring stiffness values of EOSB2, denoted as, , and, are determined as follows.

When separately applying , , and to EOSB2, the x-translational, z-translational, and y-rotational displacements of EOSB2 are given by



(8a)

Similarly, when separately applying , , and to EOSB1, the x-translational, z-translational, and y-rotational displacements of EOSB1 are given by



(8b)

, , and(Equation 8a) and , , and(8b) are then mapped into the subspace spanned by the first triplet of vibration modes giving , , and of EOSB2 and , , and of EOSB1 (Equation 6). Because the ratios of , , and of EOSB1 to , , and of EOSB2, respectively, are equal to *x*, *z*, and ** (Figure 2), the three spring stiffness of EOSB2 are obtained as



(9)

Once, , andare found (Equation 9), the stiffness matrix of EOSB2 is determined. The damping matrix of EOSB2 is then computed based on the assumption of Rayleigh damping, in which the damping ratios of the first two vibration modes of the EOSB2 are set equal to those of the corresponding vibration modes of the target building. As a result, the analytical model of EOSB2 can be completely determined.

The remaining task in designing TSMD2 is to determine the optimal values of the tuning parameters ** and *f*. The method for determining the optimal values of ** and *f* is available at Lin (2017). Because TSMD2 aims to suppress the vibrations of the target building in inelastic states, the frequency response functions of the target building in inelastic states are more adequate than those of the target building in elastic states. Nevertheless, the frequency response functions of the target building in inelastic states, which are related to the stiffness matrix of the yielded target building in the physical space, are difficult to obtain. The vibrations of EOSB2 represent those of the inelastic first triplet of vibration modes of the target building in the modal space. Therefore, the frequency response functions of EOSB2, instead of those of the target building are used for determining the optimal values of ** and *f*.



Figure 3. Sketch of the plan layout of the analytical model of an EOSB

The procedures ofdesigning a pair of elastic TSMDs for seismic control of inelastic asymmetric-plan buildings are summarized as below:

Step 1: Compute the values of *i*, *mxi*, *mzi*, *Ii*, and *si*, where *i* = 1 to 3, by performing eigenvalue analysis to the target building.

Step 2: The mass matrix, , and stiffness matrix, , of EOSB1 are obtained from Lin (2017). By assuming Rayleigh damping and giving the damping ratios of the first two modes of the first triplet of vibration modes of the target building, the damping matrix, , of EOSB1 is determined.

Step 3: The optimal values of tuning parameters ** and *f* are determined by using equations shown in Lin (2017) when the tuning parameter **\*, which is related to the mass ratio, is specified by the designer.

Step 4: With , , ,**\*,** and *f* (Steps 2 and 3), TSMD1 is completely determined by using the equations shown in Lin (2017), i.e., , , and  are attained.

Step 5: Compute the collective modal inertia force vectors **s***x*, **s***x*, and **s**** (Equation 4).

Step 6: By separately applying **s***x*, **s***x*, and **s**** to the target building, the three collective force–deformation relationships, i.e., *Dx* vs. *Ax*, *Dz* vs. *Az*, and *D* vs. *A*, are obtained from Equation 5.

Step 7: The three collective force–deformation relationships (Step 6) are presented as three bilinear curves. The post-yielding stiffness ratios of these three bilinear curves, denoted as *x*, *z*, and **, respectively, are thus attained.

Step 8: With, , and  (Step 2), the analytical model of EOSB1 (Figure 3) is constructed by means of Equation 7. In other words, the spring stiffness, damping coefficients, and the eccentricities shown in Figure 3 are determined.

Step 9: Compute the three spring stiffness of the analytical model of EOSB2, i.e., ,, and, by using Equation 9.

Step 10: With the stiffness eccentricities (Step 8) and the spring stiffness (Step 9), the stiffness matrix of EOSB2 is attained accordingly.

Step 11: The mass matrix of EOSB2 is the same as that of EOSB1. By assuming Rayleigh damping, the damping matrix of EOSB2 is determined.

Step 12: With the specified tuning parameter ** for TSMD2, the optimal values of tuning parameters ** and *f* for TSMD2 are determined by using the equations shown in Lin (2017). Note that the amplitudes of the frequency response functions presently used are those for the three directional displacements of EOSB2 with or without TSMD2.

Step 13: With EOSB2 (Steps 10 and 11) and its corresponding **\*, ** and *f* (Step 12), TSMD2 is completely determined.

**3. NUMERICAL VALIDATION**

***3.1 Selected Building Model and Ground Motion Records***

The 20-story two-way asymmetric-plan steel buildings, denoted as ASY20 (Figure 4), which was investigated in previous research (Lin 2017), are used in this study. The previous study (Lin 2017) assumed that all materials in the structural members of ASY20 remain elastic, irrespective of the ground motion intensity. In the present study, ASY20 may become inelastic during large ground motions. The full-scale structural models rather than simplified stick models were used in all the nonlinear dynamic analyses of ASY20. All the beams and columns of ASY20 are represented using beam-column elements with concentrated plasticity simulated as plastic hinges at the two ends of each element. The PISA3D structural analysis program (Lin et al. 2009) was used for the nonlinear dynamic analyses. Rigid diaphragms were assumed for all floors. The geometric nonlinearity, i.e., the P- effect, was not included in the dynamic analysis. The material nonlinearity, inherent damping, existing eccentricity, and mass dampers, which were all incorporated in the full-scale structural models, are elaborated as below.

The example building, ASY20, which is an alteration of the symmetrical prototype building located in Los Angeles and used in the SAC steel research project (FEMA 2000), is a typical office building situated on stiff soil. The seismic force-resisting system of ASY20 is the perimeter steel moment frames. The center of rigidity (CR) of each floor of ASY20 is at the geometric center. The center of mass (CM) of each floor of ASY20 is purposely moved away from the CR to result in 20% eccentricity ratios in both the x and z directions (Figure 4a). The materials used in the beams and columns of ASY20 are Dual A36 Gr. 50 steel and A572 Gr. 50 steel, respectively. The simulated stress–strain relationships of the steel materials are bilinear and have a Young’s modulus of *E* = 2*.*0 × 105 MPa and a 3% post-yielding stiffness ratio. The yield strengths of the two steel materials used in beams and columns are 340 and 345 MPa, respectively. Rayleigh damping with 5% damping ratios for the first two vibration modes of ASY20 is assumed.



Figure 4. (a) Typical floor plan and (b) elevation of ASY20

Two pairs of ground motion records (Figure 5), which were used in previous research (Lin 2017), were also considered in this study. Figure 6 shows ** = 5% damped pseudo-acceleration spectra of LA03/LA04 and LA23/LA24. The design response spectra with 72-year and 2475-year return periods for stiff soil sites in Los Angeles are also shown in Figure 6. Two intensity levels for both LA03/LA04 and LA23/LA24 were considered for ASY20. The two intensity levels were determined by the pseudo-spectral accelerations of LA03 and LA23 in the first vibration period of ASY20, denoted as *Sa*(*T1*), scaled to 0.1 g and 0.3 g. The scaling factors for LA04 and LA24 were the same as those used for LA03 and LA23, respectively.



Figure 5. Selected ground motion records (a) LA03, (b) LA04, (c) LA23, and (d) LA24



Figure 6. ** = 5% damped pseudo-acceleration spectra of LA03/LA04 and LA23/LA24 and the design response spectra with 72-year and 2475-year return periods

***3.2 Pair of elastic TSMDs for ASY20***

To construct the pair of elastic TSMDs for ASY20, the collective forces **s***x*, **s***z*, and **s**** (Equation 4) were separately applied on ASY20. Figure 7 shows the collective force–deformation relationships of ASY20. In terms of the bilinear representation of these collective force–displacement relationships (Figure 7), the post-yielding stiffness ratios *x*, *z*, and ** (Figure 2) are calculated as 0.218, 0.186, and 0.593, respectively. Besides the pushover curves, Figure 7 shows the distributions of plastic hinges at the end of each pushover analysis. The plastic hinges are denoted as red circles in Figure 7. The TSMD used in previous research (Lin 2017) is used as one of the pair of TSMDs for ASY20 in this study. The mass ratio ** was selected as 0.05 for TSMD2. Figure 8 shows the values of the controlled target *CT* (Equation A4) corresponding to different combinations of tuning parameters ** and *f*. It is found that the optimal values of ** and *f* are 2.75 and 0.84, respectively, and the corresponding *CT* value is 0.655 (Figure 8). The design of TSMD2 is eventually obtained and the properties are shown in Table 1.

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Figure 7. Collective force–displacement relationships of ASY20: (a) Ax versus Dx, (b) Az versus Dz, and (c) A versus *D*.



Figure 8. Values of the controlled target *CT* corresponding to different combinations of ** and *f* while optimizing EOSB2 for ASY20

Table 1. Mass, damping, and stiffness matrices of TSMD2 for ASY20 (units: kN, m, s).

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | |  | | |  | | |
| 554 | 0 | 0 | 113.67 | 0 | -350.3 | 259.3 | 0 | -1597 |
| 0 | 554 | 0 | 0 | 116 | 364.9 | 0 | 270.1 | 1663.8 |
| 0 | 0 | 1.07×105 | -350.3 | 364.9 | 125340 | -1597 | 1663.8 | 521455 |

***3.3 Seismic performance of ASY20 with zero, one, and two TSMDs***

Figure 9 shows the x- and z-directional peak inter-story drift ratios (IDRs), and the peak inter-story twists, denoted as y. The peak inter-story twist of the *j*th floor is defined as the peak value of the difference between the y-rotational displacement of the *j*th floor and that of the (*j*-1)th floor during a specific ground motion. For brevity, the maximums of the peak IDRs along the building height in the x- and z-directions are denoted as IDRx,max and IDRz,max, respectively. In addition, the maximum of the peak inter-story twists along the building height is denoted as y,max. Figures 9a–9c show that the values of IDRx,max, IDRz,max, and y,max of ASY20 without TSMD are 0.72%, 1%, and 0.064%, respectively. These values imply that ASY20 was slightly damaged under the excitation of LA03/LA04 scaled to *Sa*(*T1*) = 0.1 g. Under the excitation of this earthquake event whose return period is about 72 years, using one or two TSMDs is rather effective for suppressing IDRs and y, based on the values of IDRx,max/IDRz,max/y,max, being reduced to 0.66%/0.71%/0.037% and 0.65%/0.66%/0.034%, respectively (Figs. 9a–9c). Meanwhile, for the case where LA03/LA04 scaled to *Sa*(*T1*) = 0.3 g and LA23/LA24 scaled to *Sa*(*T1*) = 0.1 g are applied, the values of IDRx,max/IDRz,max/y,max of ASY20 without TSMD are 2.8%/3.9%/0.23% (Figs. 9d–9f) and 1.5%/2.6%/0.12% (Figs. 9g–9i), respectively, which implies that the building is substantially damaged. Under such intensified ground shaking, using one or two TSMDs is still effective for reducing IDRs and y (Figs. 9d–9i) but the effectiveness is not as substantial as that for the small earthquake event (i.e., Figs. 9a–9c). For LA23/LA24 scaled to *Sa*(*T1*) = 0.3 g, the values of IDRx,max/IDRz,max/y,max of ASY20 without TSMD reach 4.5%/8.5%/0.6% (Figs. 9j–9l), which implies that the building is almost completely damaged. Due to the relatively large pseudo-acceleration values of LA24 for the vibration periods *T1*, *T2*, and *T3* (Figure 6), the peak inter-story drifts in the z-direction are much larger than those in the x-direction when LA23/LA24 was applied to ASY20. In such a devastating earthquake event, the effectiveness of using one or two TSMDs on reducing IDRs and y is diminished (i.e., Figs. 9j–9l). Overall, Figure 9 indicates that the peak IDRs and the peak y of ASY20 are effectively reduced by employing TSMDs. This effectiveness is further strengthened by using a pair of elastic TSMDs instead of using one TSMD. In addition, seismic effectiveness decreases as the building deforms from an essentially elastic state to a completely damaged state.



Figure 9. (a) x-directional, (b) z-directional peak inter-story drift ratios, and (c) the peak inter-story twists of ASY20 under the excitation of LA03/LA04 scaled to *Sa*(*T1*) = 0.1 g; (d)–(f) are those under the excitation of LA03/LA04 scaled to *Sa*(*T1*) = 0.3 g; (g)–(i) and (j)–(l) are those under the excitation of LA23/LA24 scaled to *Sa*(*T1*) = 0.1 g and 0.3 g, respectively.

**4. Conclusions**

This study assessed the seismic effectiveness of TSMDs for inelastic two-way asymmetric-plan buildings. In addition, a pair of elastic TSMDs were developed, which aim to compensate for the decreased seismic effectiveness of a TSMD due to detuning effects caused by the yielding of the target building. The two TSMDs, i.e., TSMD1 and TSMD2, are designed to suppress the vibrations of a target building in elastic and inelastic states, respectively. TSMD1 is optimized from EOSB1, which reflects the collective vibration of the first triplet of vibration modes of the elastic building in the modal space. TSMD2 is optimized from EOSB2, which reflects the collective vibration of the softened first triplet of vibration modes of the inelastic building in the modal space. The inelastic force–deformation relationships of the building, which are obtained from pushing the building through the collective modal inertia force vectors of the first triplet of vibration modes, are employed to construct EOSB2. One 20-story example building was investigated. The numerical results confirmed the advantages of using TSMDs for protecting the example building against earthquakes with intensity varying from moderate to severe. In addition, using a pair of TSMDs further strengthened the seismic effectiveness by reducing the displacement demands of the example building, compared with using one TSMD. As expected, this seismic effectiveness was decreased as the building deformed from essentially elastic states to completely damaged states.

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