**Evolutionary power spectral model for the fully non-stationary ground motions and its engineering application**

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**Abstract**

Based on the power spectral model of filtered Gaussian white noise which is in the stationary earthquake process, the parametric model of fully non-stationary time-varying power spectrum is developed for describing time-varying spectral energy, and a model parameter with engineering significance is obtained through parameter identification. Firstly, the use of bedrock filter to modify the K-T spectrum makes the time-varying spectral parameters have a clear physical meaning and which is more consistent with the physical characteristics of the bedrock ground motion. Furthermore, the time-frequency distribution of the seismic is acquired on account of the generalized harmonic wavelet transform, and the target time-varying power spectrum of the actual ground motion is estimated by the Spanos-Tratskas method. At last, genetic algorithms and second optimization technology are introduced to fit mathematical expressions of time-varying parameters, and a fully non-stationary time-varying power spectral model of ground motion is established. The Japanese KIK-net strong earthquake database is adopted to identify the model parameters, which is reclassified according to the site category in China. Finally, the validity and applicability of the model are verified by the spectral representation method. The ground motion random model proposed in this paper provides a more accurately reference for describing the earthquake, and also provides a basis for the simulation of fully non-stationary ground motion.

*Keywords: fully non-stationary; time-varying power spectrum model; generalized harmonic wavelet transform; time-varying parameters; bedrock filter; K-T spectrum*

**1. INTRODUCTION**

Ground motion caused by earthquakes is a complex stochastic process whose response to the structure is highly dependent on the strength and frequency non-stationary of the ground motion. Beck [1] and Wang [2] believe that the damage of structure caused by "instantaneous resonance" phenomenon from the non-stationary ground motion is far beyond the damage of structure caused by strong earthquake. Due to the limited record of strong earthquakes and limited spectral characteristics, researching a model which is capable of simulating the time-frequency characteristics of real ground vibrations has become an important part of the stochastic dynamic response research of structures.

The ground motion non-stationary model includes both a source-based model and a site-based model. Seismologists are often interested in source-based models, which primarily describing the physics behind earthquake occurrence. The model is often lack of widespread application in actual engineering, because of the lack of relevant data such as focal mechanism, seismic wave propagation path and site conditions. Engineers pay more attention to the site-based model, describing the ground motion by fitting the target ground motion record under known site conditions. The existing site-based ground motion non-stationary models can be divided into four categories: (1) Filter white noise model. Rezaeian [3] proposes a modulation filtering Gaussian white noise process that can completely separate the intensity and frequency non-stationary characteristics. To some extent, the model can reflect the non-stationary characteristics of the actual ground motion, but it uses linear analysis when considering parameters such as the dominant frequency of the earthquake in order to simplify the model. (2) Filtered Poisson pulse model. Cornell [4] proposes a series of Poisson pulse to obtain the ground motion random process through linear filter. Although the model is non-stationary, it doesn't well fit the real ground motion record. (3) Autoregressive moving average model (ARMA). Researches [5][6] proposed a method to establish a random ground motion model using time-varying ARMA sequences, but the time-varying parameters in the model have no physical meaning. Based on the actual seismic acceleration, Conte [7] proposed a time-varying ARMA model with physical meaning by combining the Kalman filtering scheme with the window shifting method. (4) Spectral representation method. Using various evolutionary spectra to simulate ground motion has become a hot topic of research [8]. Conte and Peng [9] successfully extended the Thomson spectral estimation method to the field of non-stationary spectral analysis, and they proposed a completely non-stationary model based on the short-time multi-window spectrum estimation method. Vlachos [10] introduced spatial analysis, and then developed a bimodal evaluated spectrum model from the energy point of view. However, the error of parameter identification is increased due to the conversion from time domain to energy domain. Zhong Ting [11] constructed a new bimodal time-varying power spectrum model. However, the model considered that the bedrock spectrum is a fixed filtering white noise process, and the model ignored the bedrock spectrum time-varying characteristics.

In this paper, the time-varying characteristics of bedrock spectral parameters and Kanai-Tajimi spectral parameters are given based on the stationary power spectrum model proposed by Li Hongjing [12]. In this paper, a completely non-stationary power spectrum model with clear physical meaning is proposed. The validity and applicability of the evolutionary power spectrum model are verified by the spectral representation of the time-frequency non-stationary vibration [13].

**2. COMPLETELY NON-STATIONARY POWER SPECTRUM MODEL**

Due to the complexity of the time-frequency variation of ground motion, researchers used to generate a Gaussian stationary stochastic process based on the power spectrum model, then the non-stationary characteristic was obtained by the intensity envelope function, and finally they get the simulated ground motion. In the study of the stationary process, the Kanai-Tajimi spectral model proposed by Kanai [14] and Tajimi [15] is most widely used, which is as shown in equation (1):

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|  | (1) |

Where wg and ξg are the natural frequency and damping ratio of the site soil, respectively; and So is the power spectral density of the white noise excitation at the bedrock. The model takes into account the influence of site conditions on the ground motion power spectrum. It is considered that the cover soil layer is a linear filter system with a single degree of freedom. The seismic power spectral density is regarded as a filtered white spectrum process with clear physics significance. However, the model does not take into account the filtering characteristics of the bedrock.

Taking into account this shortcoming, Li Hongjing [12] proposed an improved power spectrum model, which corrected the high frequency part of the bedrock disturbance and limited the bedrock energy. The model is more fitted with the physical characteristics of bedrock ground motion, as shown in equation (2):

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|  | (2) |

Where *w*l and *w*h are the spectral parameters which are controlling the low and high frequencies at the bedrock, respectively. The model can reduce the high-frequency components through the bedrock filter at the bedrock. For the ground motion record with rapid drop after the acceleration peak, the descent speed of acceleration value can be adjusted by adjusting the high-frequency parameters of the model, forming a more actual earthquake recording.

Based on the power spectrum model proposed by Li Hongjing and the time-varying correction method proposed by Dedatis [16], this paper proposes a completely non-stationary time-varying power spectrum model, as shown in equations (3) and (4):

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|  | (3) |
|  |  |
|  | (4) |

Where *S*xx(w, t) is an evolutional power spectrum function which is related to frequency w and time t; *H*2(w, t) is a bedrock filter function changed with time; *w*g(t ), *ξ*g(t), *S*o(t) are the characteristic frequency, damping ratio, and intensity of white noise in time-varying models, respectively; *w*l(t) and *w*h(t) is the spectral parameter in the time-varying bedrock filter. In this paper, it is considered that seismic waves have time-varying characteristics in the filtration of bedrock and site.

**3. SEISMIC WAVE DATABASES AND THE PROCESSING**

The seismic data in this paper comes from the KIK-net strong earthquake database of the Japan Institute of Disaster Prevention Science and Technology. Japan's seismic code classifies the site category according to the characteristic period of the soil layer, while China divides the category of the building site according to the equivalent shear wave velocity and the thickness of the site cover. According to the distribution of soil layers under the stations and the information of P and S waves given in the KIK-net database, this paper reclassifies the site categories according to GB50011-2010. In order to prove that the model is applicable to most seismic waves, a total of 800 seismic records with obvious intensity and frequency non-stationary characteristics are selected from the KIK-net database, including 200 seismic waves for each site. The magnitude and epicentral distance distribution are shown in Figure 1.

When the strong earthquake instrument records the ground motion of the earthquake, the serpentine motion of the recording paper and the instability of the amplifier cause small amplitude of the rocking misalignment of the baseline, resulting in non-zero residual velocity and displacement. To eliminate this unreasonable phenomenon, this paper uses the baseline correction method [17] proposed by Osaki Shunyan to process the original ground motion record.

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| Figure 1. Distribution of magnitude and epicentral distance in records |

**4. ESTIMATION OF NON-STATIONARY POWER SPECTRUM**

In this paper, the time-frequency distribution of the ground motion signal is obtained by wavelet transform using the generalized harmonic wavelet with tightly supported frequency domain and slower time domain attenuation. According to the Spanos-Tratskas [18] method, the mean square value of the wavelet transform coefficient is transformed into the instantaneous power spectral density values of seismic ground motion.

The mother wavelet expression form [19] of generalized harmonic wavelets is:

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|  | (5) |

Where *i* is the scale subscript; *k*=0,1,..., n-m-1 is the translation transformation factor; *G* is the generalized; Δ*w*=2π/*T*0=2*w*u/N is the frequency domain sampling interval of the signal ; *w*u is the cutoff frequency of the signal; N is the number of signal sampling points; *T*0=NΔ*t* is the signal duration; Δ*t* is the sampling interval; niΔ*w* and miΔ*w* are the upper and lower limits of the frequency domain of wavelets at scale *i*.

The generalized harmonic wavelet does not require the width of each frequency band to be equal. In order to simplify the calculation, the frequency band width of the wavelet at each scale is ΔΩi=(ni-mi)Δ*w*, i=1, 2, ... , N/2(nm) is considered equal, that is ni-mi=nj-mj=n-m, and ni-1=mi, n-m=8. The generalized harmonic wavelet of the form (5) is expressed in the frequency domain as:

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|  | (6) |

The relationship between the random power spectrum estimate and the wavelet transform coefficients obtained according to the Spanos-Tratskas method [20] is as follows:

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|  | (7) |

Where (*m*j, *n*j) is the scale factor of GHW; *k* is its time shift factor; *W*ψ[(*m*j, *n*j), *k*] is a non-stationary stochastic process GHW transform coefficients at corresponding scales and times, and *m*jΔ*w*<*w*j≤*n*jΔ*w*, *kT*0/(*n*-*m*)<*t*k≤(*k*+1)*T*0/(*n*-*m*).

**5. MODEL PARAMETER IDENTIFICATION**

The time-frequency analysis of the actual seismic record is performed by the generalized harmonic wavelet transform to obtain the wavelet coefficient. The time-varying power spectrum of the target ground motion is estimated by equation (7). The global optimal estimate of positive parameters set *X*t={*w*l(t), *w*h(t), *w*g(t)*ξ*g(t), *S*0(t)} in the ground motion record is obtained based on least squares method, expressed as:

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|  | (8) |

Where *S*XX(s-t)(*w*,*t*) is the time-varying evolutionary spectral estimate of the example ground motion record obtained by the generalized harmonic wavelet transform and the Spanos-Tratskas method in Section 2, *S*XX(*w*,*t*,*X*t) is a parameterized time-varying evolution power spectrum model proposed in equation (3). *T*d is the total duration of the target ground motion record, *t*[0, *T*d].

Due to the large amount of data recorded by seismic acceleration, the application of general nonlinear optimization algorithms for the identification of least squares parameters depends on the determination of the initial value, and it is easy to fall into the local optimal solution. Considering that the genetic algorithm is a global optimization method without the prior value in the optimization process, and the method can find the global optimal value in many local optimizations, effectively on dealing with complex nonlinear problems. This paper selects the genetic algorithm to identify the time-varying spectral parameters of the model.

Using genetic algorithm to perform nonlinear least squares identification for each time point of time-varying power spectrum estimation *S*XX(s-t)(*w*,*t*) of ground motion, the instantaneous parameter value *X*t at each time point is obtained. The damping ratio of the layer is generally considered to be constant. This paper takes the average value of the damping ratio at each time point of the strong earthquake recording, namely:

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|  | (9) |

Where indicates an average. Then, the *ξ*g obtained by equation (9) is taken back to equation (8) as a known quantity, and the parameter set *X*t is again optimized by genetic algorithm to obtain the final parameter set *X*t. ={*w*l(t), *w*h(t), *w*g(t), *ξ*g, *S*0(t)}.

Obviously, except for the four parameters of the site damping ratio *ξ*g, the scatter values change with time. In order to obtain the time-varying power spectrum of the simulated ground motion, it is necessary to establish a mathematical expression function model related to the time t for the time-varying parameter. Due to the randomness of the ground motion, the parameter value mutation occurs at some time points. In order to improve the accuracy of the time-varying parameter function model, the time-varying parameters are smoothed.

Most of the previous studies have chosen the second Gaussian distribution function to fit the time-varying parameters, although the function can show the time-varying trend of the parameter value *w*l(t) to a certain extent, but considering the fitting result. The accuracy depends largely on the selection of the initial values of the secondary parameters. The fitting results are not universal. In this paper, the cubic polynomial is used to fit the time-varying parameter set *X*t={*w*l(t), *w*h(t), *w*g(t), *S*0(t)}, can well fit the trend of parameter values with time, as follows:

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|  | (10) |

Where {a1, a2, a3, a4} are the secondary parameters in the cubic polynomial, and x(t) represents different time-varying parameters in time-varying parameter set Xt={wl } (t), wh(t), wg(t), S0(t)}.

**6. EXAMPLES**

In this paper, a seismic wave record in the EW direction of the AKTH08 station in the KIK-net database is selected for sample parameter identification. The acceleration time history curve is shown in Figure2. The generalized harmonic wave transform and the Spanos-Tratskas method are used to obtain the normalized real earthquake. The dynamic acceleration time-varying power spectrum is shown in Figure3.

The genetic algorithm is used to fit the time-varying power spectrum of the target with the time-varying power spectrum proposed in this paper to obtain the time-varying parameter set *X*t, and the identified parameter set *X*t is brought into equation (3) to obtain ance fitted normalized time-varying power spectrum model. The value of the site damping ratio *ξ*g obtained by referring to equation (9) is as follows:

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|  | (12) |

The *ξ*g after taking the mean value is brought back to the equation (3), and the genetic algorithm is used to fit the target time-varying power spectrum to obtain the time-varying parameter set *X*t={*w*(TF164)(t), *w*h(t), *w*g(t), *S*0(t)}. Figure4, 5, 6, and 7 show the parameter values for each time point of the cubic polynomial fitting, and Table 1 shows the model parameters of the time-varying parameter set *X*t. Figure8 is the cubic polynomial and the site damping ratio *ξ*g based on the parameters *w*l(t), *w*h(t), *w*g(t), *S*0(t). The obtained normalized model time-varying power spectrum can be seen to better represent the time-frequency non-stationary characteristics of ground motion. Figure9 and 10 show the instantaneous power spectra normalized at 30s and 60s, respectively, to further verify the accuracy of the model power spectrum.

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| Figure 2. Acceleration of the simple ground motion | | | | | | |
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| Figure 3. The actual time-varying power spectrum | | | | | | |
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| Figure 4. Fitting of Parameters wl(t) | | | | Figure 5. Fitting of Parameters wh(t) | | |
|  | | | |  | | |
| Figure 6. Fitting of Parameter s0(t) | | | | Figure 7. Fitting of Parameters wg(t) | | |
|  | | | | | | |
| Figure 8. Simulation of power spectrum from the model | | | | | | |
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| Table 1. Model *w*l(t), *w*h(t), *w*g(t), *S*0(t) parameter values | | | | | | |
|  | a1 | a2 | | | a3 | a4 |
| *w*l | 13.641 | 0.907 | | | 0.023 | 1.356\*10-4 |
| *w*h | 14.878 | -0.409 | | | 0.010 | -5.523\*10-5 |
| *w*g | -0.040 | 0.343 | | | -0.006 | 3.061\*10-5 |
| *S*0 | 6.812 | 0.193 | | | -0.004 | 2.451\*10-5 |
|  | | |  | | | |
|  | | |  | | | |
| Figure 9. Value of power spectrum at t=30s | | | Figure 10. Value of power spectrum at t=60s | | | |

In order to verify the validity of the parameterized model power spectrum in describing the completely non-stationary characteristics of ground motion, the simulation of ground motion samples is realized by spectral representation method.

Since the computer cannot process continuous data, the model power spectrum needs to be discretized. First, the time domain of the sample ground motion is discretized:

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|  | (13) |
|  |  |
|  | (14) |

Where Δ*t* is a constant time interval and *T*d is the expected duration of the sample seismic wave.

The frequency domain is discretized with a constant frequency step Δw:

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| --- | --- |
|  | (15) |
|  |  |
|  | (16) |
|  |  |
|  | (17) |

Where *N* is the total number of frequency points, *w*u is the cutoff circle frequency of the model power spectrum, and *f*u is the Nyquist frequency.

The discrete form of the power spectrum of the model (3) is finally obtained as follows:

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| --- | --- | --- |
|  | (18) | |
|  | |  |
|  | | (19) |

According to equations (3) and (18), a power spectrum model containing 17 time-varying characteristic parameters of ground motion is obtained, and the required model parameters are combined to form a parameter set:

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| --- | --- |
|  | (20) |

Where {*Pwl*, *P*wh, *P*s0, *P*wg} are the set of coefficients fitted by the cubic polynomial {*a*1, *a*2, *a*3, *a*4}. Finally, the spectral representation method generates a simulated completely non-stationary process, namely:

|  |  |
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|  | (21) |

Where *w*n=*n*Δ*w*, *n*=0,1,...,(*N*-1), *S*XX(*w*j, *t*i) is the discretized model time-varying power spectrum obtained by equation (18), Фn is a random phase angle uniformly distributed over the interval [0, 2π].

The final simulated acceleration time history is shown in Figure11. Figure12 shows the cumulative energy curves of the 100 simulated accelerations generated by the model power spectrum. It can be seen that the cumulative energy recorded by the real ground motion is well contained in the energy curve of the simulated ground motion.

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| Figure 11. Acceleration of the simulation ground motion |
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| Figure 12. Normalized cumulative energy curve |

**7. CONCLUSION**

In this paper, the bedrock filter with time-varying characteristics is introduced to modify the traditional time-varying Kanai-Tajimi spectral model, and a parameterized ground motion time-varying power spectrum model is established. The time-frequency distribution of ground motion is obtained by generalized harmonic wavelet transform. The time-varying target power spectrum estimation is obtained by Spanos-Tratskas method. The genetic algorithm and quadratic optimization technique are used to identify the model parameters, and the mathematical expressions related to time are established.

In this paper, according to China's seismic code, the site classification of Japan's KIK-net strong earthquake database is re-divided, so that the KIK-net database has practical guiding significance for China's engineering practice. For each type of site, 200 strong earthquake records with obvious intensity and frequency non-stationary characteristics were selected, and baseline correction was performed. The parameters were identified according to the model.

A strong earthquake ground motion record of the KIK-net database is selected as an example. The validity and versatility of the modified time-varying Kanai-Tajimi spectral model are proved by the parameter identification of the power spectrum of the model and the spectral representation of the ground motion record. It can describe the non-stationary characteristics of the intensity and frequency of the ground motion more accurately. This model is suitable for seismic records of most databases and provides an important reference for ground motion input of PEER analysis.

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